

# Harmony II

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22 April 2013

# keycor

keycor h://chopin/preludes -s -l --errors

prelude28-01.krn: C Major  
prelude28-02.krn: A Minor  
prelude28-03.krn: G Major  
prelude28-04.krn: E Minor  
prelude28-05.krn: D Major  
prelude28-06.krn: B Minor  
prelude28-07.krn: A Major  
prelude28-08.krn: F# Minor  
prelude28-09.krn: E Major  
prelude28-10.krn: C# Minor  
prelude28-11.krn: B Major  
prelude28-12.krn: G# Minor  
prelude28-13.krn: F# Major  
prelude28-14.krn: E- Minor  
prelude28-15.krn: **G# Minor** X:D- major  
prelude28-16.krn: B- Minor  
prelude28-17.krn: A- Major  
prelude28-18.krn: F Minor  
prelude28-19.krn: E- Major  
prelude28-20.krn: C Minor  
prelude28-21.krn: **B- Minor** X:B- major  
prelude28-22.krn: G Minor  
prelude28-23.krn: F Major  
prelude28-24.krn: D Minor

-s == simple weights  
--aa == Aarden-Essen weights  
--bb == Bellman-Budge weights  
--kk == Krumhansl-Kessler weights  
--kp == Kostka-Payne weights  
  
-l == show filenames  
--errors == mark errors

<http://extras.humdrum.org/man/keycor>

# Hummel preludes, Op. 67

[keycor h://hummel/preludes --bb -l --errors](http://keycor.h://hummel/preludes--bb-l--errors)

prelude67-01.krn: C Major

prelude67-02.krn: **D Minor** X:A minor

prelude67-03.krn: G Major

prelude67-04.krn: E Minor

prelude67-05.krn: D Major

prelude67-06.krn: B Minor

prelude67-07.krn: A Major

prelude67-08.krn: F# Minor

prelude67-09.krn: E Major

prelude67-10.krn: C# Minor

prelude67-11.krn: B Major

prelude67-12.krn: G# Minor

prelude67-13.krn: F# Major

prelude67-14.krn: E- Minor

prelude67-15.krn: D- Major

prelude67-16.krn: B- Minor

prelude67-17.krn: A- Major

prelude67-18.krn: F Minor

prelude67-19.krn: E- Major

prelude67-20.krn: C Minor

prelude67-21.krn: B- Major

prelude67-22.krn: G Minor

prelude67-23.krn: F Major

prelude67-24.krn: D Minor



J.N. Hummel

**Allegro**  
*p*  
*armonioso*

*cresc. e sempre più pressante*  
*f*

**Meno allegro**  
*p*  
*espress.*  
*sf*  
*allegro*  
**Andante**  
*p*  
*f*

# Krumhansl-Schmuckler key-finding algorithm

- The best key assignment is the key which generates the highest Pearson correlation between a pitch histogram extracted from the music compared to major/minor key prototypes

Pearson correlation (normalized correlation):

$$R(x, y) = \frac{\sum (x_n - \bar{x})(y_n - \bar{y})}{\sqrt{\sum (x_n - \bar{x})^2 \sum (y_n - \bar{y})^2}}$$

# Pitch-class histogram



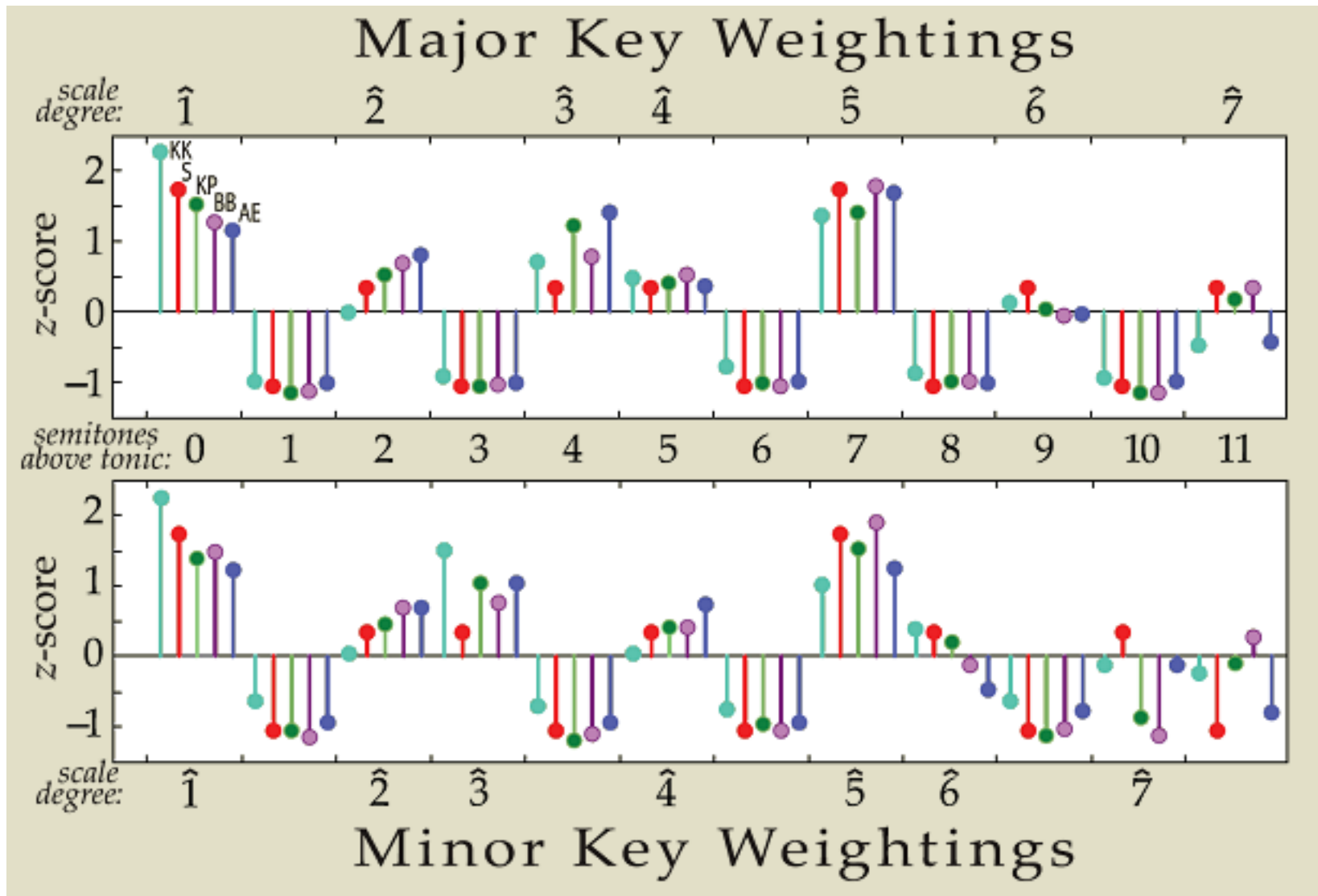
extracted histogram as a 12-dimensional vector: (8, 0, 0, 0, 2, 11, 0, 5, 7, 0, 5, 2)

# Prototype weights

| <i>scale degree</i>      | <i>semi-tones above tonic</i> | KK   | AE    | KP    | BB    | LC |
|--------------------------|-------------------------------|------|-------|-------|-------|----|
| <b>Major Key Weights</b> |                               |      |       |       |       |    |
| 1                        | 0                             | 6.35 | 17.77 | 0.748 | 16.80 | 2  |
|                          | 1                             | 2.23 | 0.15  | 0.060 | 0.86  | 0  |
| 2                        | 2                             | 3.48 | 14.93 | 0.488 | 12.95 | 1  |
|                          | 3                             | 2.33 | 0.16  | 0.082 | 1.41  | 0  |
| 3                        | 4                             | 4.38 | 19.80 | 0.670 | 13.49 | 1  |
| 4                        | 5                             | 4.09 | 11.36 | 0.460 | 11.93 | 1  |
|                          | 6                             | 2.52 | 0.29  | 0.096 | 1.25  | 0  |
| 5                        | 7                             | 5.19 | 22.06 | 0.715 | 20.28 | 2  |
|                          | 8                             | 2.39 | 0.15  | 0.104 | 1.80  | 0  |
| 6                        | 9                             | 3.66 | 8.15  | 0.366 | 8.04  | 1  |
|                          | 10                            | 2.29 | 0.23  | 0.057 | 0.62  | 0  |
| 7                        | 11                            | 2.88 | 4.95  | 0.400 | 10.57 | 1  |
| <b>Minor Key Weights</b> |                               |      |       |       |       |    |
| 1                        | 0                             | 6.33 | 18.26 | 0.712 | 18.16 | 2  |
|                          | 1                             | 2.68 | 0.74  | 0.084 | 0.69  | 0  |
| 2                        | 2                             | 3.52 | 14.05 | 0.474 | 12.99 | 1  |
|                          | 3                             | 5.38 | 16.86 | 0.618 | 13.34 | 1  |
| 3                        | 4                             | 2.60 | 0.70  | 0.049 | 1.07  | 0  |
|                          | 5                             | 3.53 | 14.44 | 0.460 | 11.15 | 1  |
| 4                        | 6                             | 2.54 | 0.70  | 0.105 | 1.38  | 0  |
|                          | 7                             | 4.75 | 18.62 | 0.747 | 21.07 | 2  |
| 5                        | 8                             | 3.98 | 4.57  | 0.404 | 7.49  | 1  |
|                          | 9                             | 2.69 | 1.93  | 0.067 | 1.53  | 0  |
| 6                        | 10                            | 3.34 | 7.38  | 0.133 | 0.92  | 1  |
|                          | 11                            | 3.17 | 1.76  | 0.330 | 10.21 | 0  |

# Prototype comparisons

Unnormalized major-key simple weights: (2, 0, 1, 0, 1, 1, 0, 2, 0, 1, 0, 1)



# KS algorithm (2)

$$R(x, y) = \frac{\sum (x_n - \bar{x})(y_n - \bar{y})}{\sqrt{\sum (x_n - \bar{x})^2 \sum (y_n - \bar{y})^2}}$$

$$\text{key}_k = \arg \max_k R(x, y_k)$$

$$\text{key}_k = \arg \max_k \frac{1}{N} \sum_n z_{x,n} z_{y_k,n}$$

dot product of  
normalized  
vectors

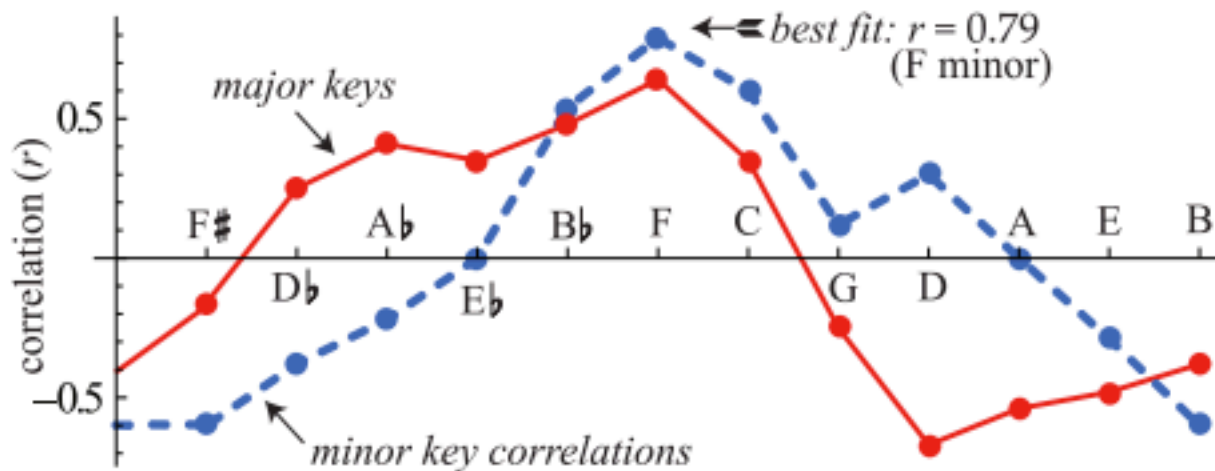


# Key correlation spectrum



pitch-class counts  
(duration weighted):

|                           |                           |                           |                           |
|---------------------------|---------------------------|---------------------------|---------------------------|
| C: 8                      | D $\sharp$ /E $\flat$ : 0 | F $\sharp$ /G $\flat$ : 0 | A: 0                      |
| C $\sharp$ /D $\flat$ : 0 | E: 2                      | G: 5                      | A $\sharp$ /B $\flat$ : 5 |
| D: 0                      | F: 11                     | G $\sharp$ /A $\flat$ : 7 | B: 2                      |



pitch-class histogram:  $x = (8, 0, 0, 0, 2, 11, 0, 5, 7, 0, 5, 2)$   
 major key prototype:  $y_M = (2, 0, 1, 0, 1, 1, 0, 2, 0, 1, 0, 1)$   
 minor key prototype:  $y_m = (2, 0, 1, 1, 0, 1, 0, 2, 1, 0, 1, 0)$

# Key correlation spectrum (2)

`keycor -a -s h://wtc/wtc2p12.krn`

The best key is: F Minor

|                       |                       |
|-----------------------|-----------------------|
| Major[0] = -0.0469801 | Minor[0] = 0.629898   |
| Major[1] = 0.664563   | Minor[1] = -0.138204  |
| Major[2] = -0.853396  | Minor[2] = -0.194762  |
| Major[3] = 0.631723   | Minor[3] = 0.356228   |
| Major[4] = -0.550534  | Minor[4] = -0.729332  |
| Major[5] = 0.372648   | Minor[5] = 0.934585   |
| Major[6] = 0.115398   | Minor[6] = -0.556007  |
| Major[7] = -0.590672  | Minor[7] = 0.0935041  |
| Major[8] = 0.808697   | Minor[8] = 0.126345   |
| Major[9] = -0.700141  | Minor[9] = -0.430119  |
| Major[10] = 0.476643  | Minor[10] = 0.774032  |
| Major[11] = -0.327949 | Minor[11] = -0.866167 |

-a == show all key weights [0] = C, [1] == C#/D-flat, etc.

# Evaluate all key-cycle pieces

- Download all key-cycle works:

```
humcat -s h://wtc h://chopin/preludes h://hummel/preludes > keycycles.krns
```

- wtc: 48 preludes, 48 fugues chopin: 24 preludes, hummel: 24 preludes: **144 pieces**
- Evaluate various key prototypes:

```
keycor -l --errors --bellman keycycles.krns | grep X: | wc -l  
10
```

```
keycor -l --errors --aarden keycycles.krns | grep X: | wc -l  
8
```

```
keycor -l --errors --kk keycycles.krns | grep X: | wc -l  
13
```

```
keycor -l --errors --simple keycycles.krns | grep X: | wc -l  
7
```

```
keycor -l --errors --kp keycycles.krns | grep X: | wc -l  
9
```

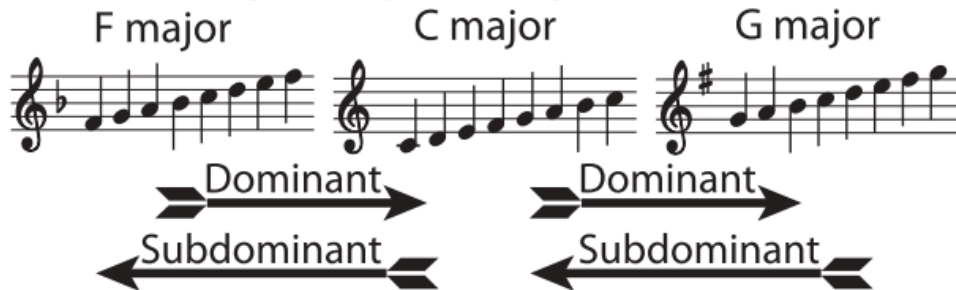
# Types of identification errors

| J.S. Bach, The Well-tempered Clavier |     |     |     |     |        |    |    |    |    |          |     |     |    |    | Chopin preludes op. 28 |    |     |    |     |   |    |    |    |    |   |       |
|--------------------------------------|-----|-----|-----|-----|--------|----|----|----|----|----------|-----|-----|----|----|------------------------|----|-----|----|-----|---|----|----|----|----|---|-------|
| Book I                               |     |     |     |     |        |    |    |    |    | Book II  |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   |       |
| preludes                             |     |     |     |     | fugues |    |    |    |    | preludes |     |     |    |    | fugues                 |    |     |    |     |   |    |    |    |    |   |       |
| key                                  | AE  | BB  | KK  | KP  | S      | AE | BB | KK | KP | S        | AE  | BB  | KK | KP | S                      | AE | BB  | KK | KP  | S | AE | BB | KK | KP | S | key   |
| C maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | C maj |
| c min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | c min |
| C#maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | C#maj |
| c#min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | c#min |
| D maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | D maj |
| d min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | d min |
| E♭maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | E♭maj |
| e♭min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | e♭min |
| E maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | E maj |
| e min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | e min |
| F maj                                | rel | rel | rel | rel | rel    |    |    |    |    |          | rel | rel |    |    |                        |    |     |    |     |   |    |    |    |    |   | F maj |
| f min                                |     |     |     |     |        |    |    |    |    |          |     | rel |    |    |                        |    |     |    |     |   |    |    |    |    |   | f min |
| F#maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | F#maj |
| f#min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | f#min |
| G maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | G maj |
| g min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | g min |
| A♭maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | A♭maj |
| a♭min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | a♭min |
| A maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    | rel |    | dom |   |    |    |    |    |   | A maj |
| a min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | a min |
| B♭maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | B♭maj |
| b♭min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | b♭min |
| B maj                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | B maj |
| b min                                |     |     |     |     |        |    |    |    |    |          |     |     |    |    |                        |    |     |    |     |   |    |    |    |    |   | b min |
| errors                               | 1   | 1   | 2   | 1   | 1      | 0  | 2  | 1  | 2  | 1        | 1   | 2   | 2  | 1  | 0                      | 2  | 0   | 3  | 1   | 0 | 2  | 4  | 4  | 3  | 2 |       |

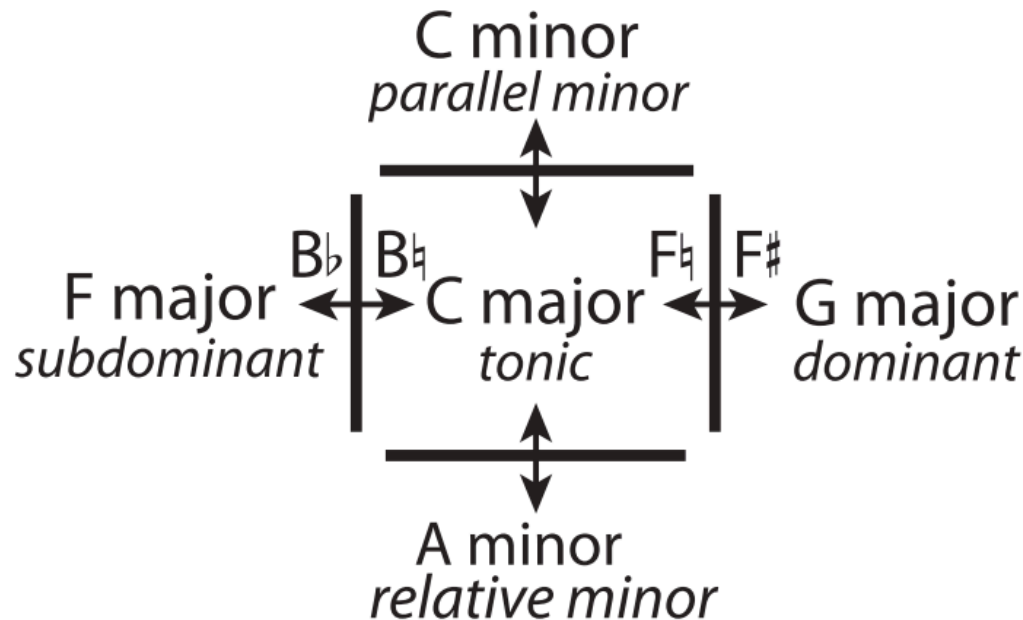
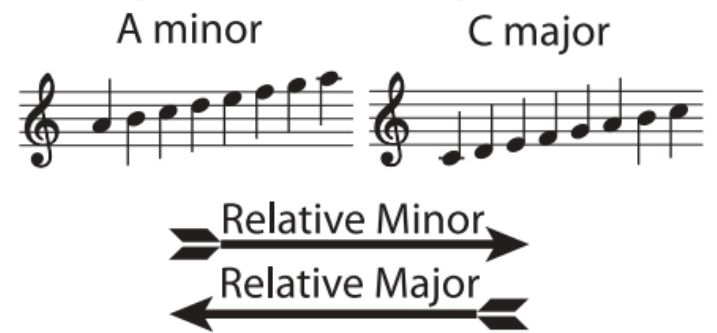
cumulative errors: AE: 6/120 (5.0%) BB: 9/120 (7.5%) KK: 12/120 (10.0%) KP: 8/120 (6.7%) S: 4/120 (3.3%)

# Key confusions

## Primary Major Key Boundaries



## Primary Modal Key Boundary



# Evaluate Beethoven piano sonatas

(102 movements)

```
keycor -l --errors -s h://beethoven/sonatas | grep X: | wc -l
```

3

```
keycor -l --errors --aa h://beethoven/sonatas | grep X: | wc -l
```

6

```
keycor -l --errors --kp h://beethoven/sonatas | grep X: | wc -l
```

5

```
keycor -l --errors --bb h://beethoven/sonatas | grep X: | wc -l
```

5

```
keycor -l --errors --kk h://beethoven/sonatas | grep X: | wc -l
```

29

# Evaluate Mozart piano sonatas

(69 movements)

```
keycor -l --errors -s h://mozart/sonatas | grep X: | wc -l
```

2

```
keycor -l --errors --aa h://mozart/sonatas | grep X: | wc -l
```

1

```
keycor -l --errors --kp h://mozart/sonatas | grep X: | wc -l
```

1

```
keycor -l --errors --bb h://mozart/sonatas | grep X: | wc -l
```

2

```
keycor -l --errors --kk h://mozart/sonatas | grep X: | wc -l
```

10

# Evaluate Scarlatti piano sonatas

(59 sonatas)

```
keycor -l --errors -s h://scarlatti/sonatas | grep X: | wc -l  
11
```

```
keycor -l --errors --aa h://scarlatti/sonatas | grep X: | wc -l  
6
```

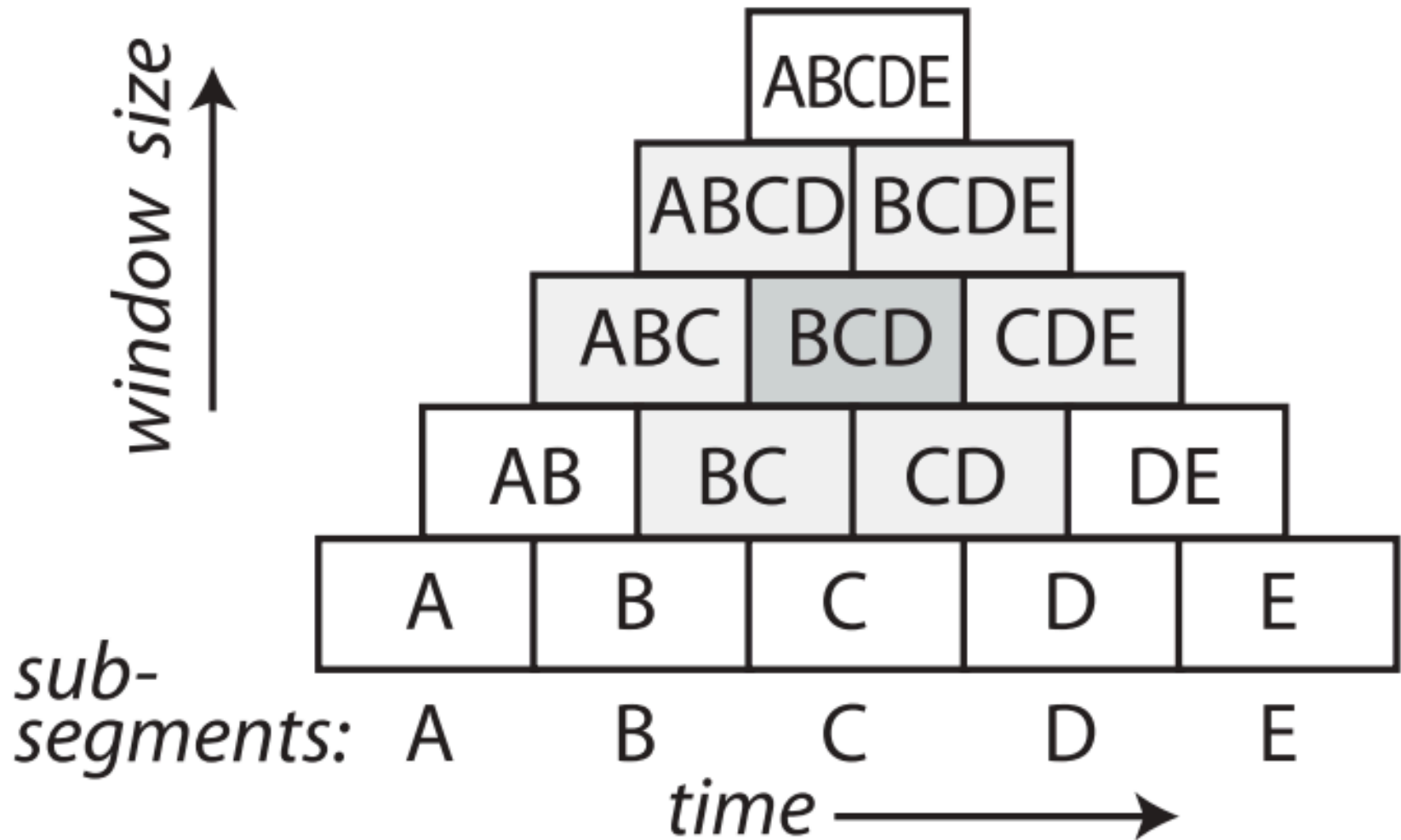
```
keycor -l --errors --kp h://scarlatti/sonatas | grep X: | wc -l  
4
```

```
keycor -l --errors --bb h://scarlatti/sonatas | grep X: | wc -l  
6
```

```
keycor -l --errors --kk h://scarlatti/sonatas | grep X: | wc -l  
26
```



# Keyscape plotting domain

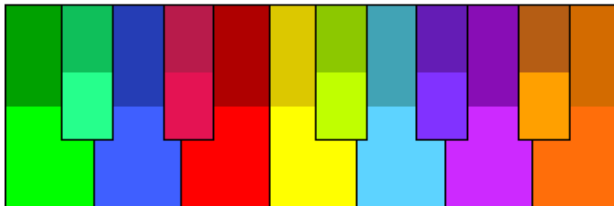
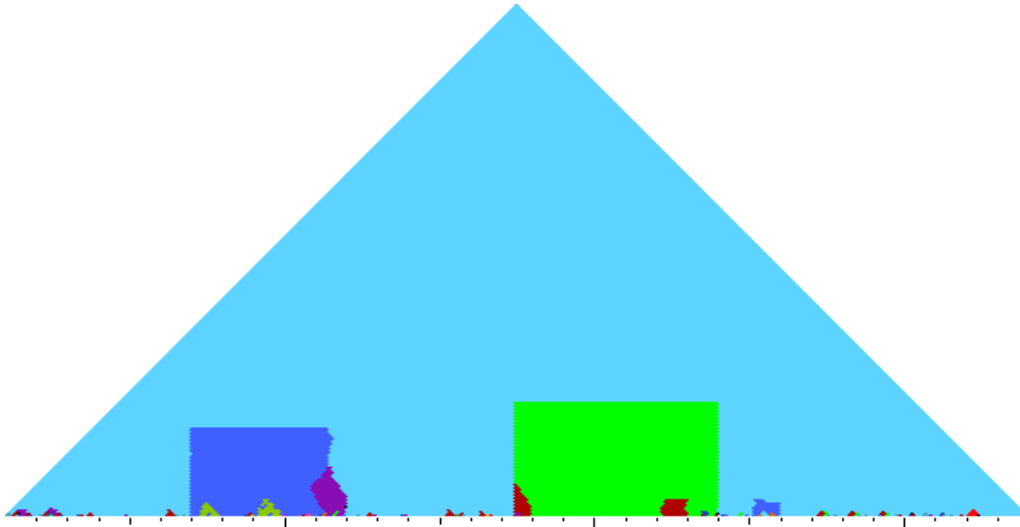


# mkeyscape

<http://extras.humdrum.org/man/mkeyscape>

mkeyscape == “MIDI-aware keyscape program” (can read MIDI files and Humdrum files)

```
mkeyscape h://chopin/preludes/prelude28-03.krn -n --trim -l --aa | convert - z.png
```



-n == show measure ticks  
--trim == post clean-up  
-l == show color mapping legend  
--aa == Use Aarden-Essen weights

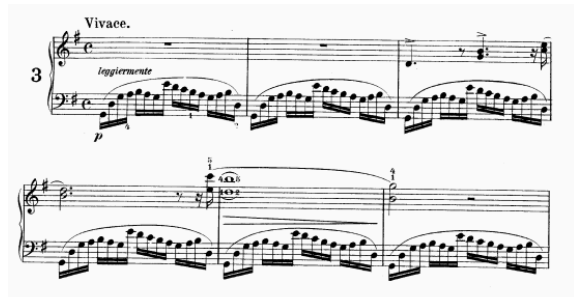
convert == ImageMagick image processing program (mkeyscape output is type PPM)

# Chopin Prelude #3, Op. 28

G major

D major

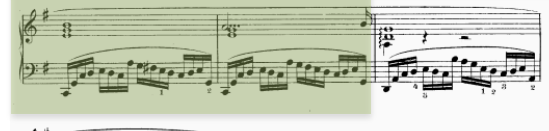
C major



D major

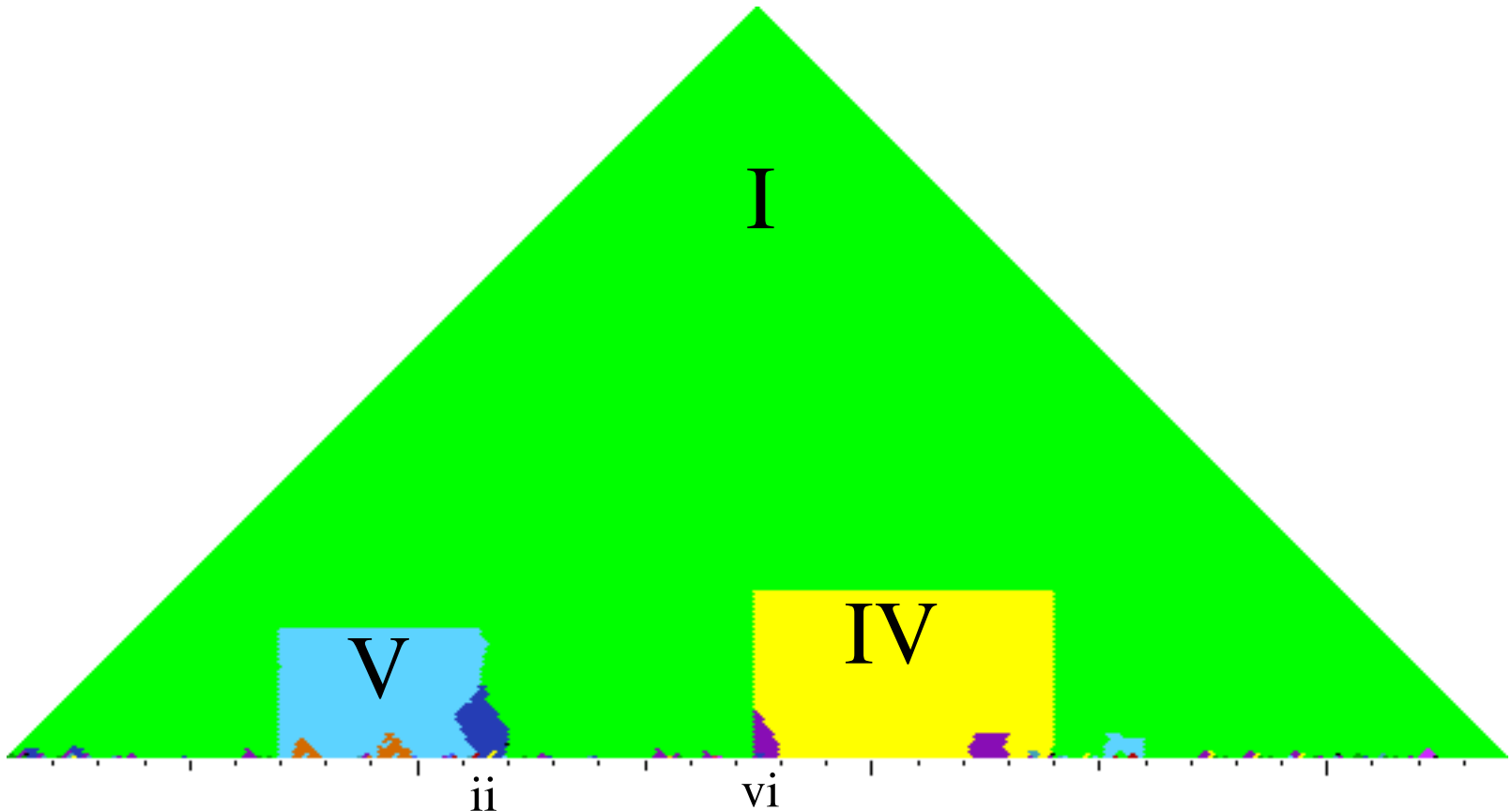


C major



# Functional coloring

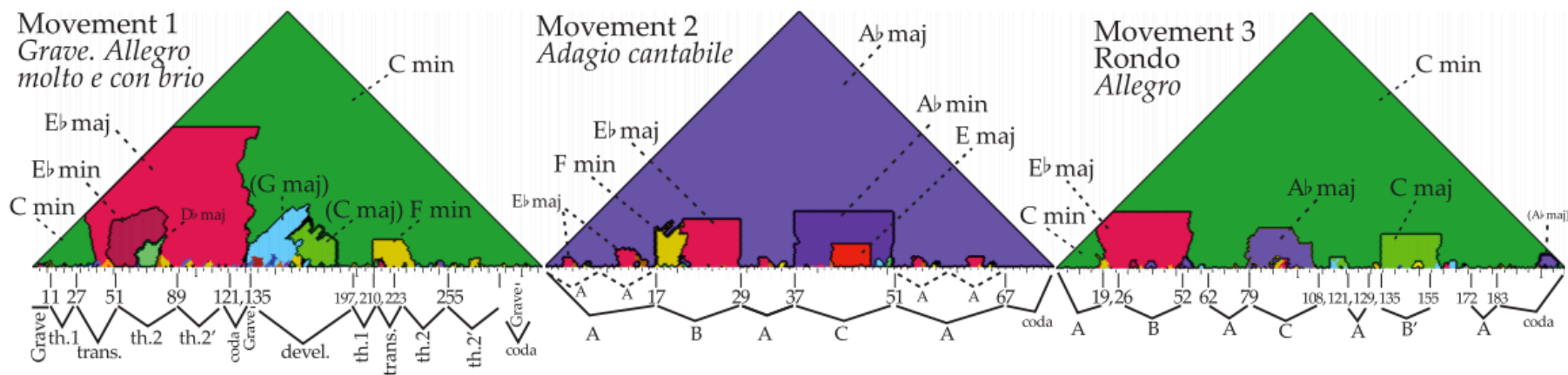
```
transpose -k C h://chopin/preludes/prelude28-03.krn | mkeyscape -n --trim --aa | convert - z.png
```



<http://extras.humdrum.org/man/transpose>

# Sonata movements

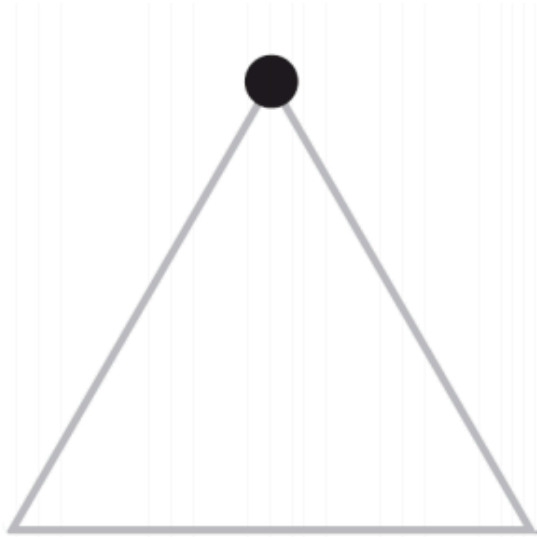
## Ludwig van Beethoven Piano Sonata no. 8 in C minor, op. 13 ("*Pathétique*")



<http://extras.humdrum.org/man/mkeyscape/beet-sonata>

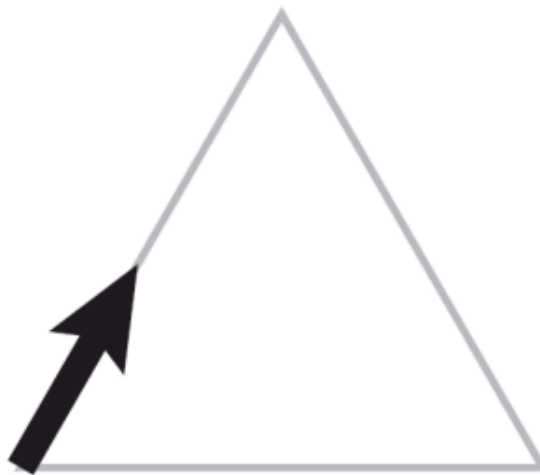
<http://extras.humdrum.org/man/mkeyscape/moz-sonata>

# Key evaluation methods



Top-Level Test

high-data behavior



Leading-Edge Test

low-data behavior

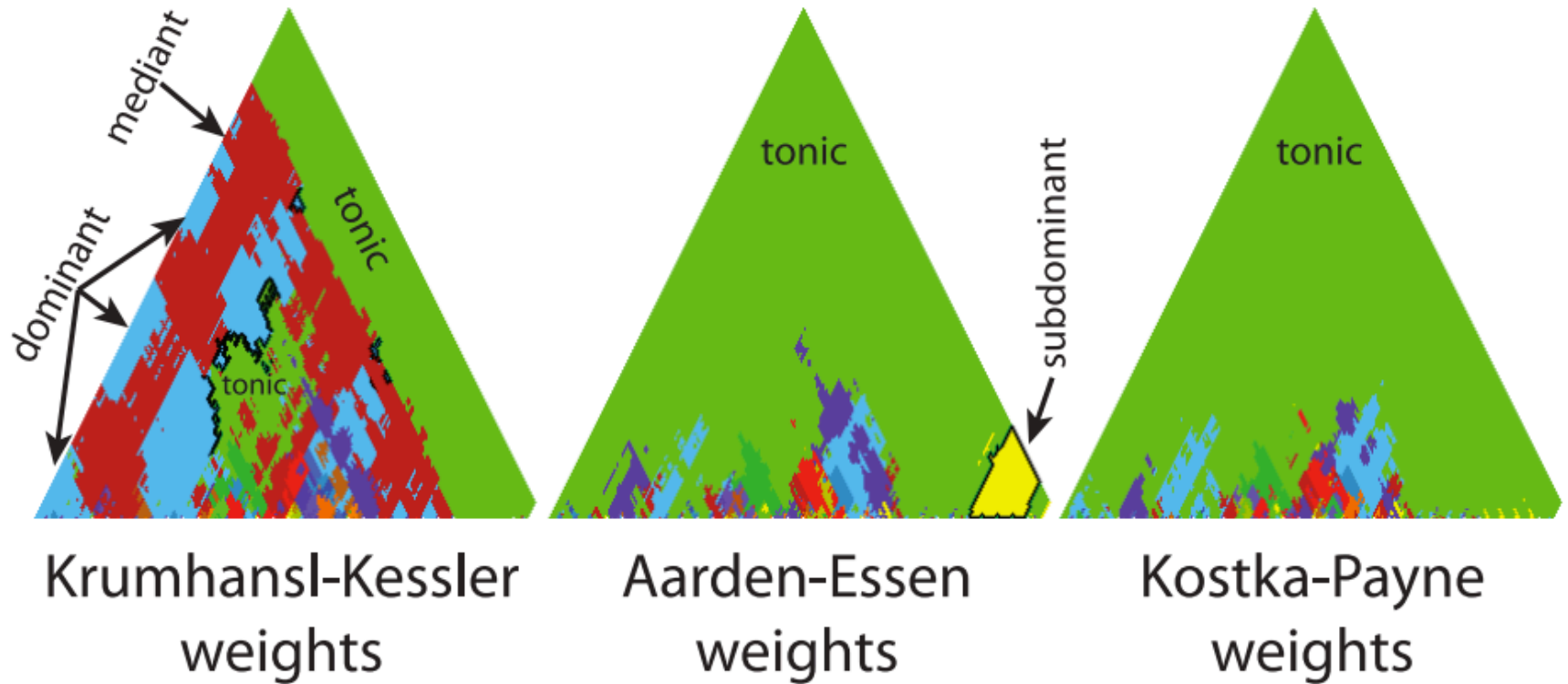
(& trailing edge)



Tonic-Area Test

intermediate behavior

# Keyscapes by weight



# Chopin prelude in C major 28/1

1

*mf*

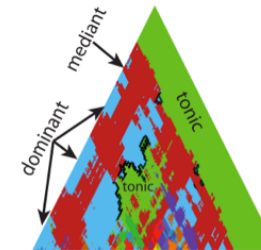
*cresc.*

*stretto*

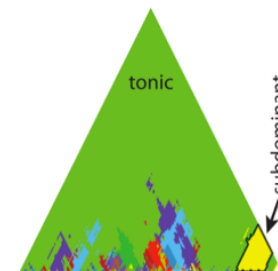
*ff*

*p*

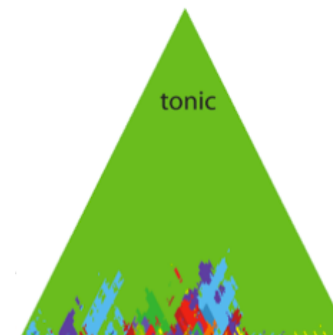
*pp*



Krumhansl-Kessler  
weights



Aarden-Essen  
weights

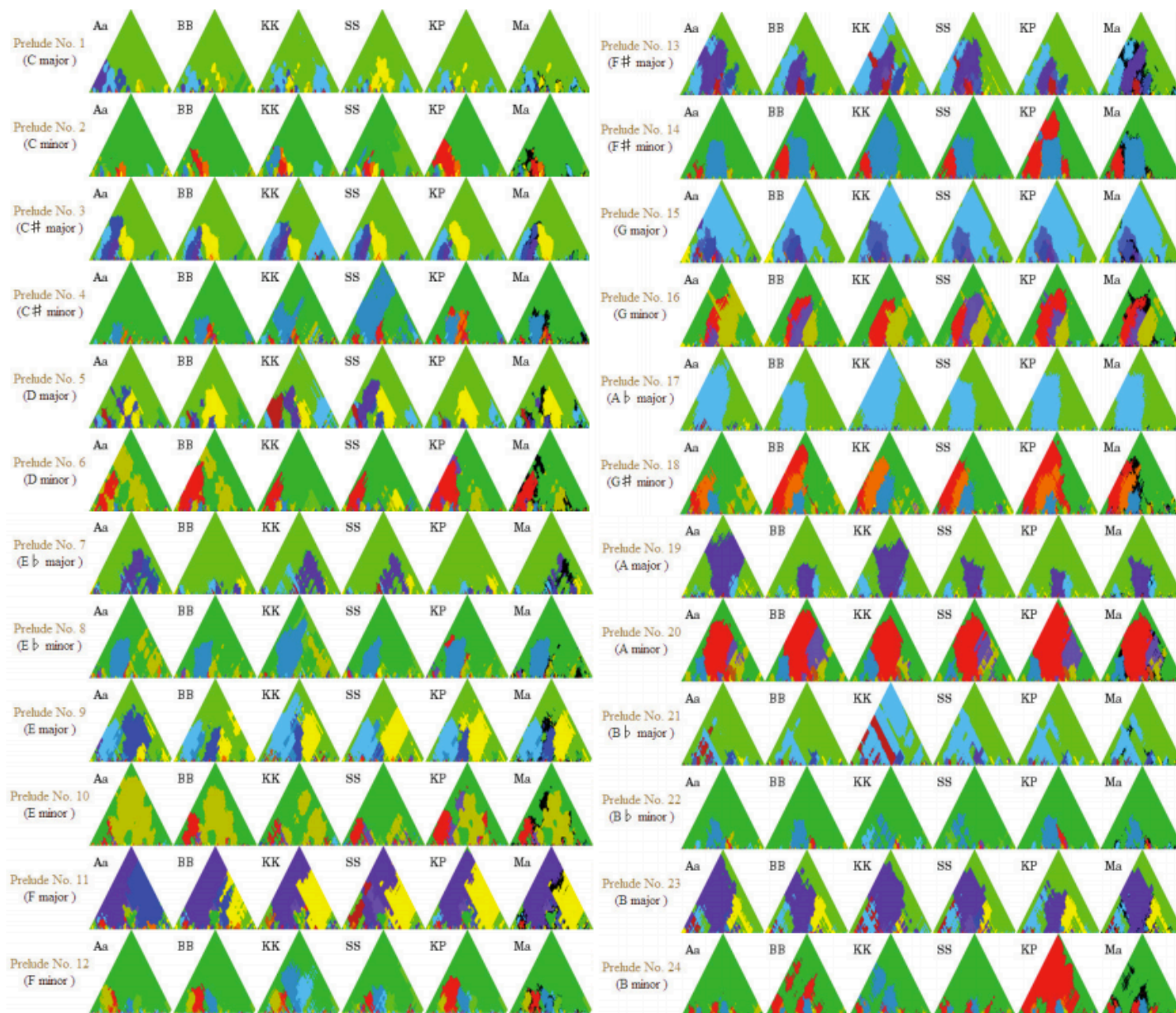


Kostka-Payne  
weights



# Keyscapes of WTC Book I preludes

<http://extras.humdrum.org/man/mkeyscape/wtc>



# Tonic-area evaluation

|       | J.S. Bach, The Well-tempered Clavier |    |    |        |    |    |          |    |    |        |    |    | Chopin preludes |    |    |        |    |    |          |    |    |        |    |    |    |    |    |    |    |    |
|-------|--------------------------------------|----|----|--------|----|----|----------|----|----|--------|----|----|-----------------|----|----|--------|----|----|----------|----|----|--------|----|----|----|----|----|----|----|----|
|       | Book I                               |    |    |        |    |    | Book II  |    |    |        |    |    | op. 28          |    |    |        |    |    |          |    |    |        |    |    |    |    |    |    |    |    |
|       | preludes                             |    |    | fugues |    |    | preludes |    |    | fugues |    |    | preludes        |    |    | fugues |    |    | preludes |    |    | fugues |    |    |    |    |    |    |    |    |
| key   | AE                                   | BB | KK | KP     | S  | M  | AE       | BB | KK | KP     | S  | M  | AE              | BB | KK | KP     | S  | M  | AE       | BB | KK | KP     | S  | M  | AE | BB | KK | KP | S  | M  |
| C maj | 74                                   | 74 | 70 | 82     | 74 | 80 | 43       | 59 | 32 | 80     | 43 | 46 | 20              | 23 | 26 | 26     | 24 | 22 | 85       | 88 | 77 | 92     | 83 | 87 | 84 | 83 | 37 | 87 | 85 | 86 |
| c min | 87                                   | 85 | 76 | 74     | 65 | 81 | 88       | 81 | 82 | 36     | 83 | 83 | 14              | 11 | 18 | 10     | 16 | 13 | 90       | 91 | 90 | 72     | 90 | 90 | 83 | 86 | 80 | 83 | 77 | 82 |
| C#maj | 63                                   | 67 | 50 | 68     | 68 | 67 | 65       | 73 | 31 | 75     | 67 | 69 | 79              | 82 | 34 | 81     | 72 | 79 | 92       | 91 | 85 | 94     | 88 | 92 | 39 | 20 | 2  | 24 | 29 | 23 |
| c#min | 86                                   | 81 | 64 | 73     | 37 | 75 | 89       | 78 | 88 | 37     | 88 | 87 | 65              | 47 | 55 | 29     | 60 | 56 | 84       | 51 | 57 | 10     | 61 | 57 | 80 | 85 | 39 | 81 | 72 | 80 |
| D maj | 64                                   | 63 | 50 | 68     | 54 | 62 | 60       | 77 | 74 | 76     | 79 | 75 | 48              | 62 | 34 | 61     | 53 | 54 | 30       | 76 | 37 | 83     | 70 | 67 | 54 | 77 | 41 | 78 | 72 | 71 |
| d min | 47                                   | 51 | 75 | 56     | 70 | 62 | 84       | 86 | 71 | 79     | 73 | 82 | 75              | 76 | 56 | 59     | 77 | 73 | 90       | 90 | 89 | 77     | 89 | 89 | 55 | 55 | 32 | 45 | 43 | 46 |
| E♭maj | 64                                   | 86 | 67 | 88     | 76 | 76 | 46       | 71 | 50 | 88     | 52 | 56 | 65              | 86 | 49 | 84     | 78 | 78 | 90       | 91 | 46 | 89     | 85 | 89 | 72 | 73 | 60 | 77 | 74 | 73 |
| e♭min | 56                                   | 72 | 33 | 64     | 71 | 65 | 78       | 75 | 80 | 26     | 83 | 76 | 61              | 43 | 37 | 27     | 57 | 47 | 38       | 25 | 30 | 9      | 65 | 31 | 59 | 60 | 59 | 52 | 42 | 55 |
| E maj | 38                                   | 51 | 28 | 39     | 41 | 39 | 53       | 74 | 46 | 89     | 62 | 65 | 52              | 68 | 45 | 65     | 66 | 62 | 70       | 83 | 37 | 92     | 57 | 72 | 11 | 13 | 8  | 14 | 41 | 12 |
| e min | 34                                   | 46 | 61 | 48     | 71 | 49 | 26       | 11 | 26 | 7      | 13 | 13 | 60              | 51 | 34 | 26     | 59 | 51 | 73       | 46 | 62 | 19     | 59 | 57 | 48 | 65 | 49 | 67 | 81 | 62 |
| F maj | 3                                    | 7  | 6  | 7      | 8  | 6  | 24       | 44 | 12 | 67     | 26 | 31 | 10              | 33 | 12 | 59     | 25 | 24 | 67       | 73 | 67 | 79     | 74 | 73 | 82 | 84 | 38 | 77 | 65 | 77 |
| f min | 79                                   | 79 | 53 | 70     | 71 | 74 | 77       | 61 | 56 | 19     | 70 | 62 | 26              | 28 | 36 | 24     | 33 | 29 | 75       | 66 | 51 | 25     | 54 | 57 | 68 | 71 | 48 | 69 | 67 | 68 |
| F#maj | 41                                   | 57 | 27 | 59     | 43 | 45 | 34       | 63 | 35 | 69     | 50 | 50 | 40              | 65 | 28 | 72     | 50 | 51 | 40       | 79 | 56 | 79     | 70 | 69 | 85 | 86 | 75 | 89 | 84 | 87 |
| f#min | 73                                   | 62 | 46 | 45     | 64 | 60 | 90       | 87 | 82 | 36     | 84 | 85 | 70              | 64 | 63 | 46     | 76 | 67 | 82       | 42 | 76 | 3      | 80 | 74 | 44 | 65 | 19 | 55 | 32 | 46 |
| G maj | 18                                   | 23 | 16 | 21     | 25 | 21 | 32       | 52 | 32 | 63     | 53 | 51 | 62              | 74 | 27 | 70     | 65 | 67 | 43       | 69 | 8  | 77     | 56 | 55 | 61 | 53 | 46 | 63 | 58 | 58 |
| g min | 46                                   | 56 | 51 | 51     | 40 | 48 | 65       | 52 | 71 | 23     | 68 | 62 | 78              | 76 | 70 | 64     | 73 | 74 | 56       | 23 | 59 | 10     | 71 | 49 | 31 | 46 | 37 | 47 | 30 | 38 |
| A♭maj | 45                                   | 65 | 35 | 57     | 64 | 56 | 44       | 78 | 47 | 81     | 71 | 70 | 32              | 57 | 30 | 55     | 43 | 44 | 78       | 81 | 77 | 78     | 77 | 79 | 23 | 24 | 34 | 33 | 51 | 32 |
| a♭min | 59                                   | 46 | 46 | 38     | 54 | 50 | 71       | 64 | 50 | 10     | 72 | 65 | 40              | 38 | 25 | 26     | 32 | 33 | 93       | 88 | 85 | 22     | 87 | 89 | 56 | 58 | 54 | 52 | 55 | 56 |
| A maj | 40                                   | 75 | 48 | 78     | 74 | 71 | 53       | 86 | 63 | 89     | 78 | 79 | 50              | 59 | 67 | 69     | 60 | 62 | 19       | 65 | 11 | 71     | 37 | 38 | 86 | 87 | 61 | 85 | 84 | 86 |
| a min | 34                                   | 22 | 38 | 18     | 29 | 27 | 69       | 27 | 57 | 4      | 66 | 55 | 73              | 72 | 56 | 56     | 62 | 67 | 77       | 67 | 79 | 22     | 75 | 75 | 16 | 24 | 17 | 20 | 16 | 21 |
| B♭maj | 74                                   | 76 | 20 | 69     | 58 | 69 | 29       | 54 | 28 | 72     | 37 | 39 | 58              | 73 | 41 | 75     | 58 | 66 | 58       | 81 | 63 | 79     | 70 | 72 | 22 | 24 | 27 | 31 | 30 | 28 |
| b♭min | 81                                   | 84 | 78 | 78     | 79 | 82 | 37       | 32 | 70 | 24     | 65 | 40 | 52              | 33 | 51 | 23     | 50 | 45 | 79       | 38 | 73 | 9      | 61 | 64 | 67 | 72 | 60 | 49 | 69 | 66 |
| B maj | 25                                   | 40 | 31 | 52     | 38 | 37 | 70       | 84 | 54 | 84     | 77 | 79 | 45              | 65 | 20 | 61     | 50 | 52 | 25       | 82 | 59 | 88     | 66 | 72 | 94 | 94 | 68 | 93 | 90 | 92 |
| b min | 84                                   | 69 | 66 | 25     | 83 | 74 | 78       | 53 | 68 | 7      | 71 | 67 | 41              | 28 | 35 | 21     | 51 | 35 | 53       | 15 | 23 | 5      | 36 | 21 | 68 | 83 | 77 | 74 | 66 | 73 |

# Tonic-area evaluation (2)

  = best result        = better than simple weights

| <i>weight set:</i>        |        | AE                                                                | BB                                                                | KK   | KP                                                                | SS   | Majority                                                          |
|---------------------------|--------|-------------------------------------------------------------------|-------------------------------------------------------------------|------|-------------------------------------------------------------------|------|-------------------------------------------------------------------|
| Major keys:               | mean   | 0.70                                                              | <span style="border: 1px dashed black; padding: 2px;">0.90</span> | 0.58 | <span style="border: 1px solid black; padding: 2px;">0.96</span>  | 0.83 | 0.82                                                              |
|                           | median | 0.73                                                              | <span style="border: 1px dashed black; padding: 2px;">0.96</span> | 0.57 | <span style="border: 1px solid black; padding: 2px;">1.00</span>  | 0.88 | <span style="border: 1px dashed black; padding: 2px;">0.88</span> |
|                           | sd     | 0.22                                                              | <span style="border: 1px dashed black; padding: 2px;">0.14</span> | 0.23 | <span style="border: 1px solid black; padding: 2px;">0.11</span>  | 0.15 | 0.16                                                              |
| Minor keys:               | mean   | <span style="border: 1px solid black; padding: 2px;">0.91</span>  | 0.81                                                              | 0.81 | 0.56                                                              | 0.87 | 0.84                                                              |
|                           | median | <span style="border: 1px solid black; padding: 2px;">0.99</span>  | 0.88                                                              | 0.84 | 0.61                                                              | 0.93 | 0.87                                                              |
|                           | sd     | <span style="border: 1px dashed black; padding: 2px;">0.14</span> | 0.21                                                              | 0.17 | 0.30                                                              | 0.14 | <span style="border: 1px solid black; padding: 2px;">0.13</span>  |
| Major and Minor combined: | mean   | 0.81                                                              | <span style="border: 1px solid black; padding: 2px;">0.86</span>  | 0.69 | 0.76                                                              | 0.85 | 0.83                                                              |
|                           | median | <span style="border: 1px dashed black; padding: 2px;">0.91</span> | <span style="border: 1px solid black; padding: 2px;">0.93</span>  | 0.72 | <span style="border: 1px dashed black; padding: 2px;">0.90</span> | 0.90 | 0.87                                                              |
|                           | sd     | 0.21                                                              | 0.18                                                              | 0.23 | 0.30                                                              | 0.14 | <span style="border: 1px solid black; padding: 2px;">0.14</span>  |

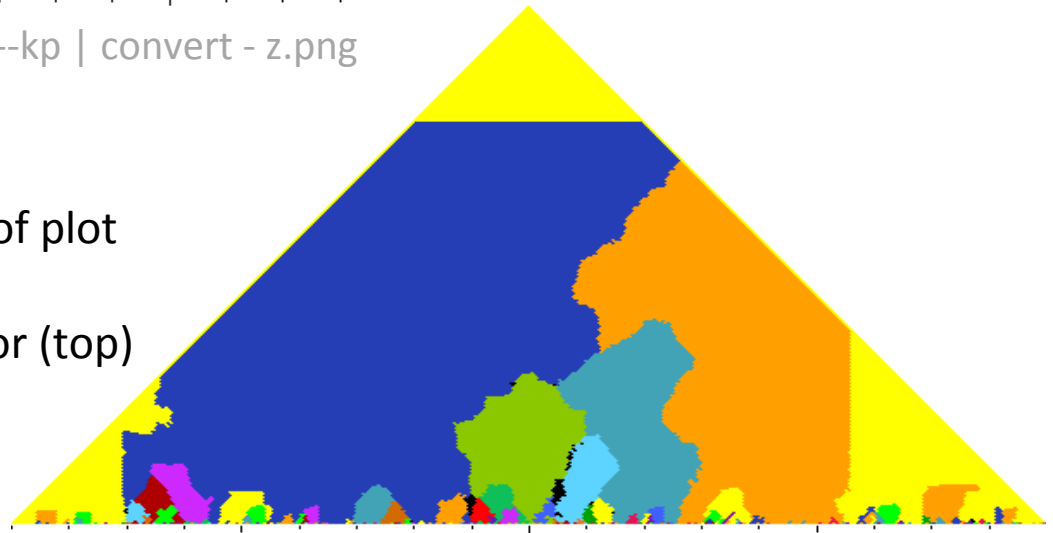
# WTC1p11 (F major prelude)



- mostly secondary keys
- 3-8% of keyscape is tonic (yellow)
- blue = D minor (relative minor)

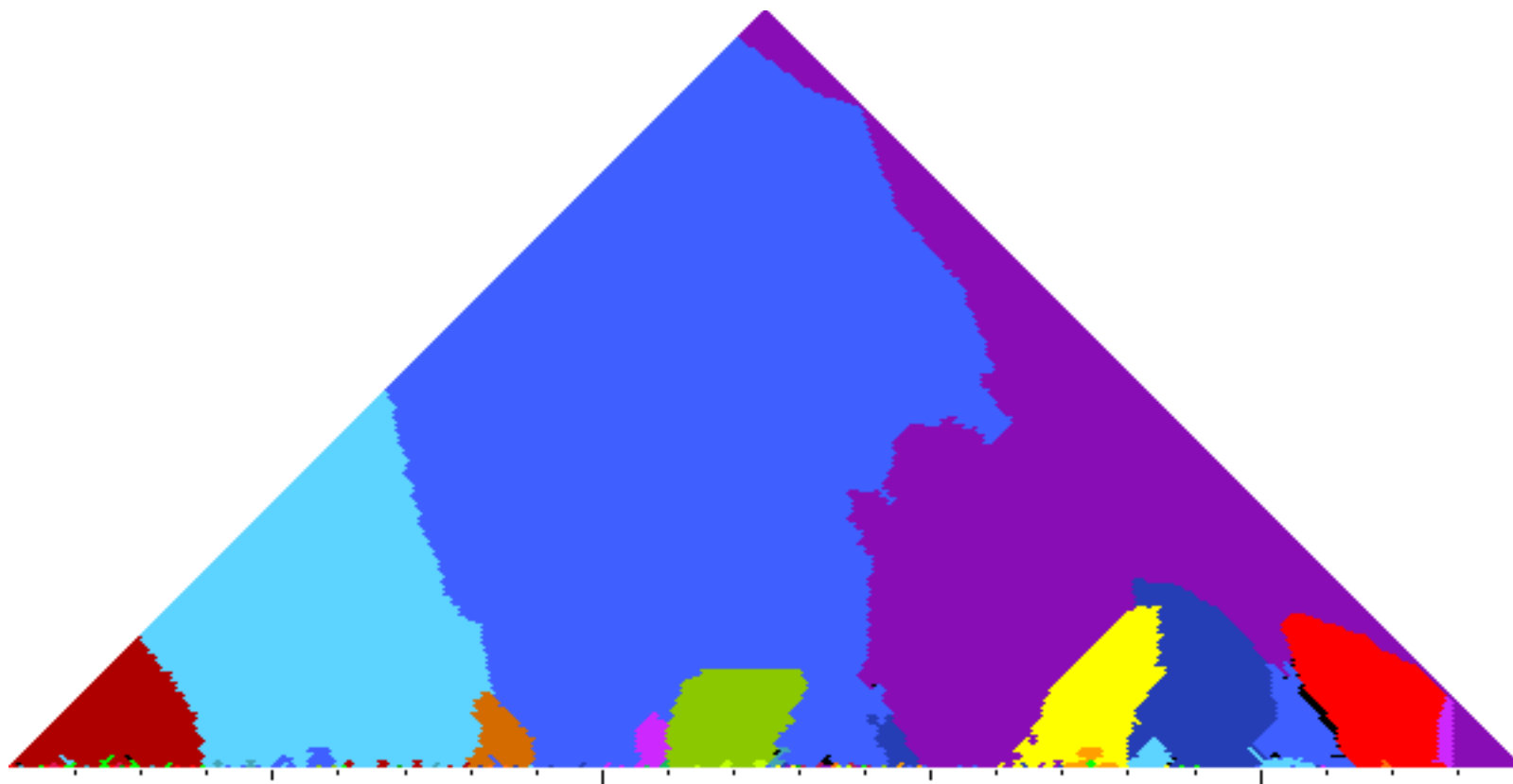
mkeyscape h://wtc/wtc1p11.krn -n --kp | convert - z.png

- post processing:
- regions anchored to bottom of plot
- tonal biasing
- key assignment is now F major (top)



mkeyscape h://wtc/wtc1p11.krn -n --kp --trim | convert - z.png

# Chopin prelude in A minor 28/2

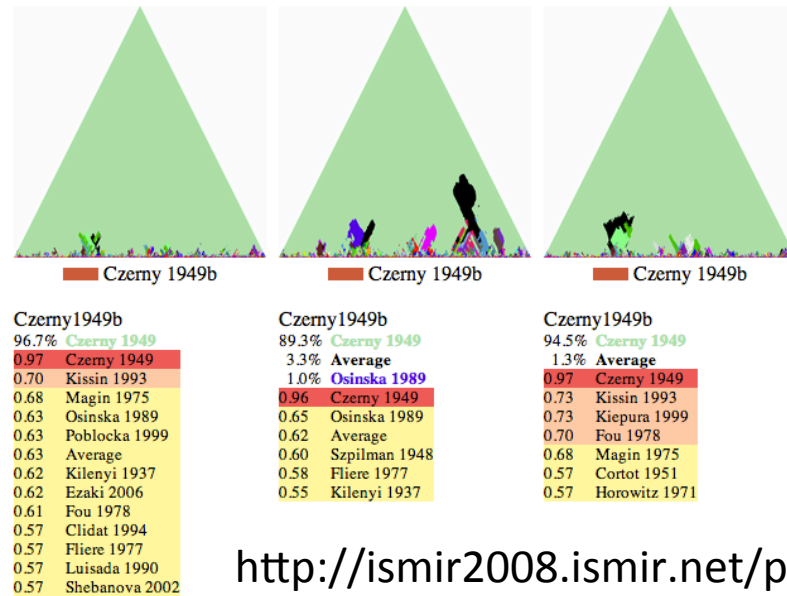


mkeyscape h://chopin/preludes/prelude28-02.krn -n --ss --trim | convert - z.png



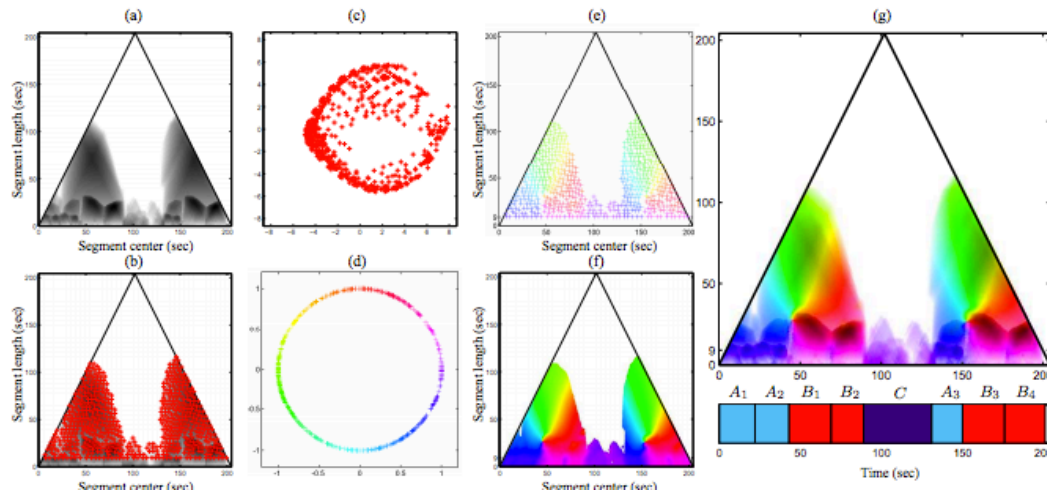
# Other Scape plotting applications

performance analysis



[http://ismir2008.ismir.net/papers/ISMIR2008\\_240.pdf](http://ismir2008.ismir.net/papers/ISMIR2008_240.pdf)

form analysis



<http://ismir2012.ismir.net/event/papers/097-ismir-2012.pdf>

# More Info

<http://purl.stanford.edu/br237mp4161>

Lots of keyscares (raw plots; not post-processed):

<https://ccrma.stanford.edu/~craig/keyscape/class>

