

Harmony II

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keycor

keycor h://chopin/preludes -s -l --errors

prelude28-01.krn: C Major
prelude28-02.krn: A Minor
prelude28-03.krn: G Major
prelude28-04.krn: E Minor
prelude28-05.krn: D Major
prelude28-06.krn: B Minor
prelude28-07.krn: A Major
prelude28-08.krn: F# Minor
prelude28-09.krn: E Major
prelude28-10.krn: C# Minor
prelude28-11.krn: B Major
prelude28-12.krn: G# Minor
prelude28-13.krn: F# Major
prelude28-14.krn: E- Minor
prelude28-15.krn: **G# Minor** X:D- major
prelude28-16.krn: B- Minor
prelude28-17.krn: A- Major
prelude28-18.krn: F Minor
prelude28-19.krn: E- Major
prelude28-20.krn: C Minor
prelude28-21.krn: **B- Minor** X:B- major
prelude28-22.krn: G Minor
prelude28-23.krn: F Major
prelude28-24.krn: D Minor

-s == simple weights
--aa == Aarden-Essen weights
--bb == Bellman-Budge weights
--kk == Krumhansl-Kessler weights
--kp == Kostka-Payne weights

-l == show filenames
--errors == mark errors

Hummel Preludes, Op. 67

[keycor h://hummel/preludes --bb -l --errors](http://keycor.h://hummel/preludes--bb-l--errors)

prelude67-01.krn: C Major

prelude67-02.krn: **D Minor** X:A minor

prelude67-03.krn: G Major

prelude67-04.krn: E Minor

prelude67-05.krn: D Major

prelude67-06.krn: B Minor

prelude67-07.krn: A Major

prelude67-08.krn: F# Minor

prelude67-09.krn: E Major

prelude67-10.krn: C# Minor

prelude67-11.krn: B Major

prelude67-12.krn: G# Minor

prelude67-13.krn: F# Major

prelude67-14.krn: E- Minor

prelude67-15.krn: D- Major

prelude67-16.krn: B- Minor

prelude67-17.krn: A- Major

prelude67-18.krn: F Minor

prelude67-19.krn: E- Major

prelude67-20.krn: C Minor

prelude67-21.krn: B- Major

prelude67-22.krn: G Minor

prelude67-23.krn: F Major

prelude67-24.krn: D Minor



J.N. Hummel

Allegro
p
armonioso

cresc. e sempre più pressante
f

Meno allegro
p
espress.
sf
allegro
Andante
p
f

Krumhansl-Schmuckler Key-Finding Algorithm

- The best key assignment is the key which generates the highest Pearson correlation between a pitch histogram extracted from the music compared to major/minor key prototypes

Pearson correlation (normalized correlation):

$$R(x, y) = \frac{\sum (x_n - \bar{x})(y_n - \bar{y})}{\sqrt{\sum (x_n - \bar{x})^2 \sum (y_n - \bar{y})^2}}$$

Pitch-class histogram



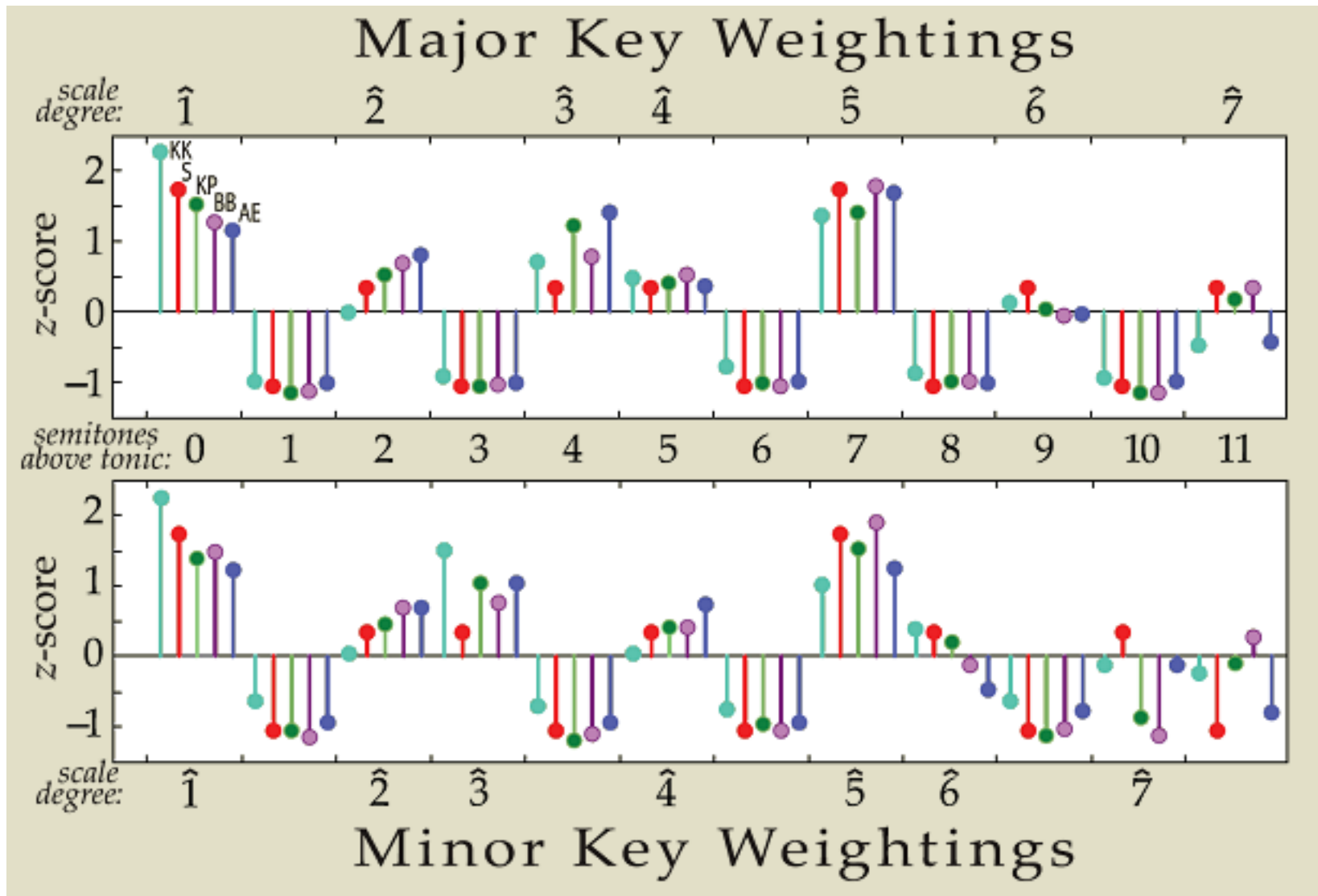
extracted histogram as a 12-dimensional vector: (8, 0, 0, 0, 2, 11, 0, 5, 7, 0, 5, 2)

Prototype weights

<i>scale degree</i>	<i>semi-tones above tonic</i>	KK	AE	KP	BB	LC
Major Key Weights						
1	0	6.35	17.77	0.748	16.80	2
	1	2.23	0.15	0.060	0.86	0
2	2	3.48	14.93	0.488	12.95	1
	3	2.33	0.16	0.082	1.41	0
3	4	4.38	19.80	0.670	13.49	1
4	5	4.09	11.36	0.460	11.93	1
5	6	2.52	0.29	0.096	1.25	0
	7	5.19	22.06	0.715	20.28	2
6	8	2.39	0.15	0.104	1.80	0
	9	3.66	8.15	0.366	8.04	1
7	10	2.29	0.23	0.057	0.62	0
	11	2.88	4.95	0.400	10.57	1
Minor Key Weights						
1	0	6.33	18.26	0.712	18.16	2
	1	2.68	0.74	0.084	0.69	0
2	2	3.52	14.05	0.474	12.99	1
	3	5.38	16.86	0.618	13.34	1
3	4	2.60	0.70	0.049	1.07	0
	5	3.53	14.44	0.460	11.15	1
4	6	2.54	0.70	0.105	1.38	0
	7	4.75	18.62	0.747	21.07	2
5	8	3.98	4.57	0.404	7.49	1
	9	2.69	1.93	0.067	1.53	0
6	10	3.34	7.38	0.133	0.92	1
	11	3.17	1.76	0.330	10.21	0

Prototype comparisons

Unnormalized major-key simple weights: (2, 0, 1, 0, 1, 1, 0, 2, 0, 1, 0, 1)



KS Algorithm (2)

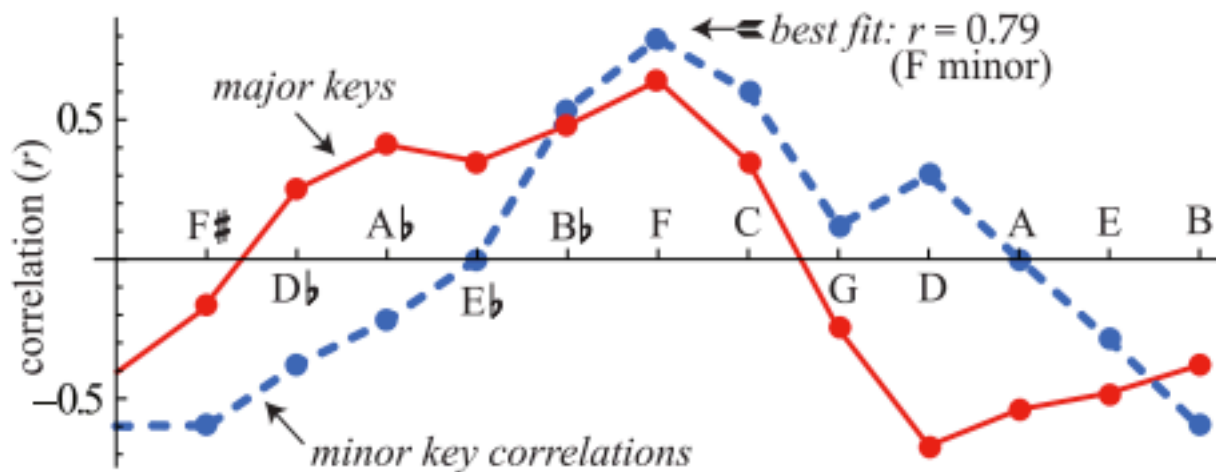
$$R(x, y) = \frac{\sum (x_n - \bar{x})(y_n - \bar{y})}{\sqrt{\sum (x_n - \bar{x})^2 \sum (y_n - \bar{y})^2}}$$

$$\text{key}_k = \arg \max_k R(x, y_k)$$

$$\text{key}_k = \arg \max_k \frac{1}{N} \sum_n z_{x,n} z_{y_k,n}$$

dot product of
normalized
vectors

Key correlation spectrum



pitch-class histogram: $x = (8, 0, 0, 0, 2, 11, 0, 5, 7, 0, 5, 2)$
 major key prototype: $y_M = (2, 0, 1, 0, 1, 1, 0, 2, 0, 1, 0, 1)$
 minor key prototype: $y_m = (2, 0, 1, 1, 0, 1, 0, 2, 1, 0, 1, 0)$

Key correlation spectrum (2)

`keycor -a -s h://wtc/wtc2p12.krn`

The best key is: F Minor

Major[0] = -0.0469801	Minor[0] = 0.629898
Major[1] = 0.664563	Minor[1] = -0.138204
Major[2] = -0.853396	Minor[2] = -0.194762
Major[3] = 0.631723	Minor[3] = 0.356228
Major[4] = -0.550534	Minor[4] = -0.729332
Major[5] = 0.372648	Minor[5] = 0.934585
Major[6] = 0.115398	Minor[6] = -0.556007
Major[7] = -0.590672	Minor[7] = 0.0935041
Major[8] = 0.808697	Minor[8] = 0.126345
Major[9] = -0.700141	Minor[9] = -0.430119
Major[10] = 0.476643	Minor[10] = 0.774032
Major[11] = -0.327949	Minor[11] = -0.866167

-a == show all key weights [0] = C, [1] == C#/D-flat, etc.

Evaluate all key-cycle pieces

- Download all key-cycle works:

```
humcat -s h://wtc h://chopin/preludes h://hummel/preludes > keycycles.krns
```

- wtc: 48 preludes, 48 fugues chopin: 24 preludes, hummel: 24 preludes: **144 pieces**
- Evaluate various key prototypes:

```
keycor -l --errors --bellman keycycles.krns | grep X: | wc -l  
10
```

```
keycor -l --errors --aarden keycycles.krns | grep X: | wc -l  
8
```

```
keycor -l --errors --kk keycycles.krns | grep X: | wc -l  
13
```

```
keycor -l --errors --simple keycycles.krns | grep X: | wc -l  
7
```

```
keycor -l --errors --kp keycycles.krns | grep X: | wc -l  
9
```

Types of identification errors

J.S. Bach, The Well-tempered Clavier															Chopin preludes op. 28															
	Book I					Book II																								
	preludes					fugues					preludes					fugues														
key	AE	BB	KK	KP	S	AE	BB	KK	KP	S	AE	BB	KK	KP	S	AE	BB	KK	KP	S	AE	BB	KK	KP	S	key				
C maj																										C maj				
c min																										c min				
C#maj																										C#maj				
c#min																										c#min				
D maj																										D maj				
d min																										d min				
E♭maj																										E♭maj				
e♭min																										e♭min				
E maj																										E maj				
e min																										e min				
F maj	rel	rel	rel	rel	rel						rel	rel														F maj				
f min												rel														f min				
F#maj																										F#maj				
f#min																										f#min				
G maj																										G maj				
g min																										g min				
A♭maj																										A♭maj				
a♭min																										a♭min				
A maj																	rel		dom							A maj				
a min																										a min				
B♭maj																										B♭maj				
b♭min																										b♭min				
B maj																										B maj				
b min																										b min				
errors	1	1	2	1	1	0	2	1	2	1	1	2	2	1	0	2	0	3	1	0	2	4	4	3	2					

cumulative errors: AE: 6/120 (5.0%) BB: 9/120 (7.5%) KK: 12/120 (10.0%) KP: 8/120 (6.7%) S: 4/120 (3.3%)

Evaluate Beethoven piano sonatas

(102 movements)

```
keycor -l --errors -s h://beethoven/sonatas | grep X: | wc -l
```

3

```
keycor -l --errors --aa h://beethoven/sonatas | grep X: | wc -l
```

6

```
keycor -l --errors --kp h://beethoven/sonatas | grep X: | wc -l
```

5

```
keycor -l --errors --bb h://beethoven/sonatas | grep X: | wc -l
```

5

```
keycor -l --errors --kk h://beethoven/sonatas | grep X: | wc -l
```

29

Evaluate Mozart piano sonatas

(69 movements)

```
keycor -l --errors -s h://mozart/sonatas | grep X: | wc -l
```

2

```
keycor -l --errors --aa h://mozart/sonatas | grep X: | wc -l
```

1

```
keycor -l --errors --kp h://mozart/sonatas | grep X: | wc -l
```

1

```
keycor -l --errors --bb h://mozart/sonatas | grep X: | wc -l
```

2

```
keycor -l --errors --kk h://mozart/sonatas | grep X: | wc -l
```

10

Evaluate Scarlatti piano sonatas

(59 sonatas)

```
keycor -l --errors -s h://scarlatti/sonatas | grep X: | wc -l  
11
```

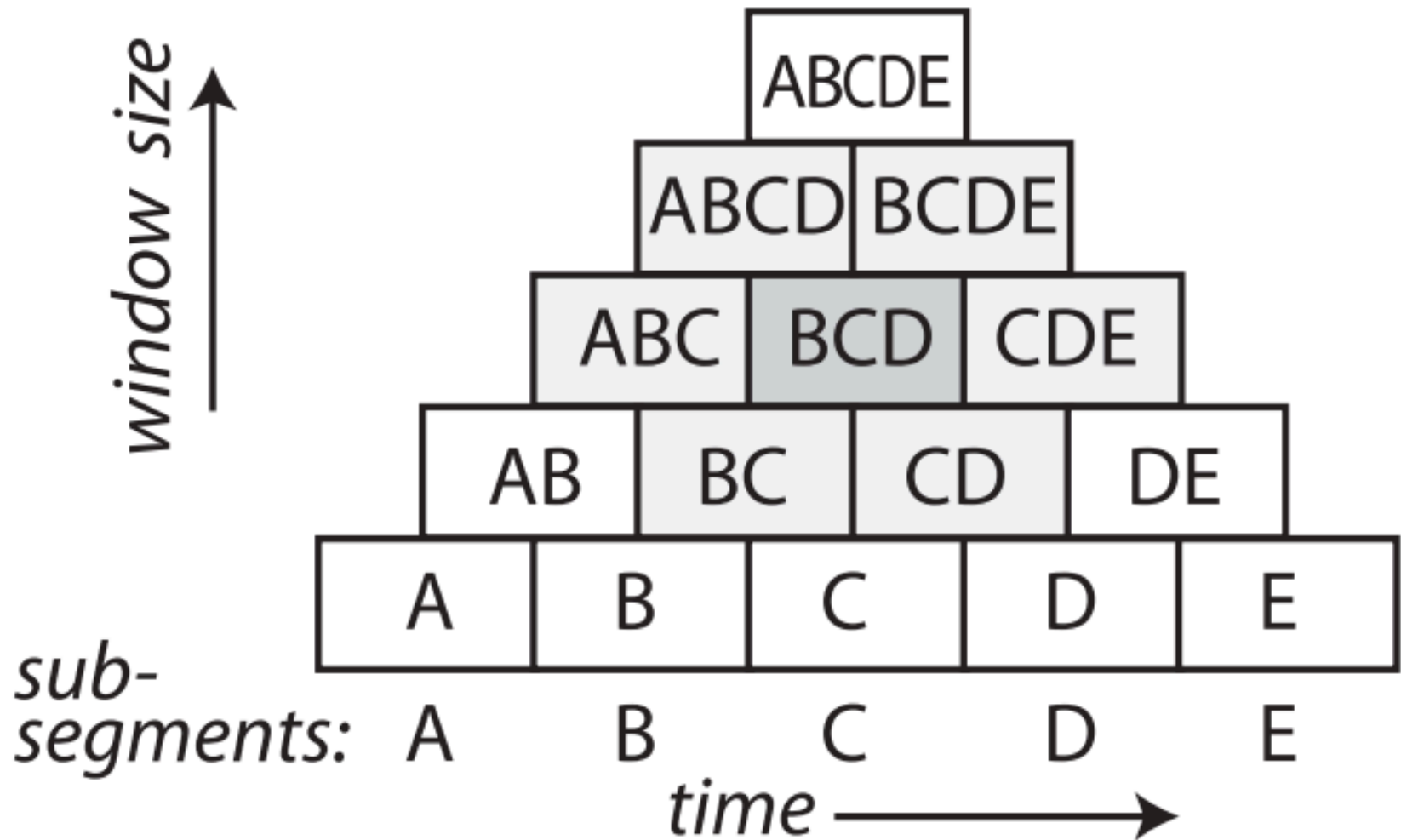
```
keycor -l --errors --aa h://scarlatti/sonatas | grep X: | wc -l  
6
```

```
keycor -l --errors --kp h://scarlatti/sonatas | grep X: | wc -l  
4
```

```
keycor -l --errors --bb h://scarlatti/sonatas | grep X: | wc -l  
6
```

```
keycor -l --errors --kk h://scarlatti/sonatas | grep X: | wc -l  
26
```

Keyscape plotting domain

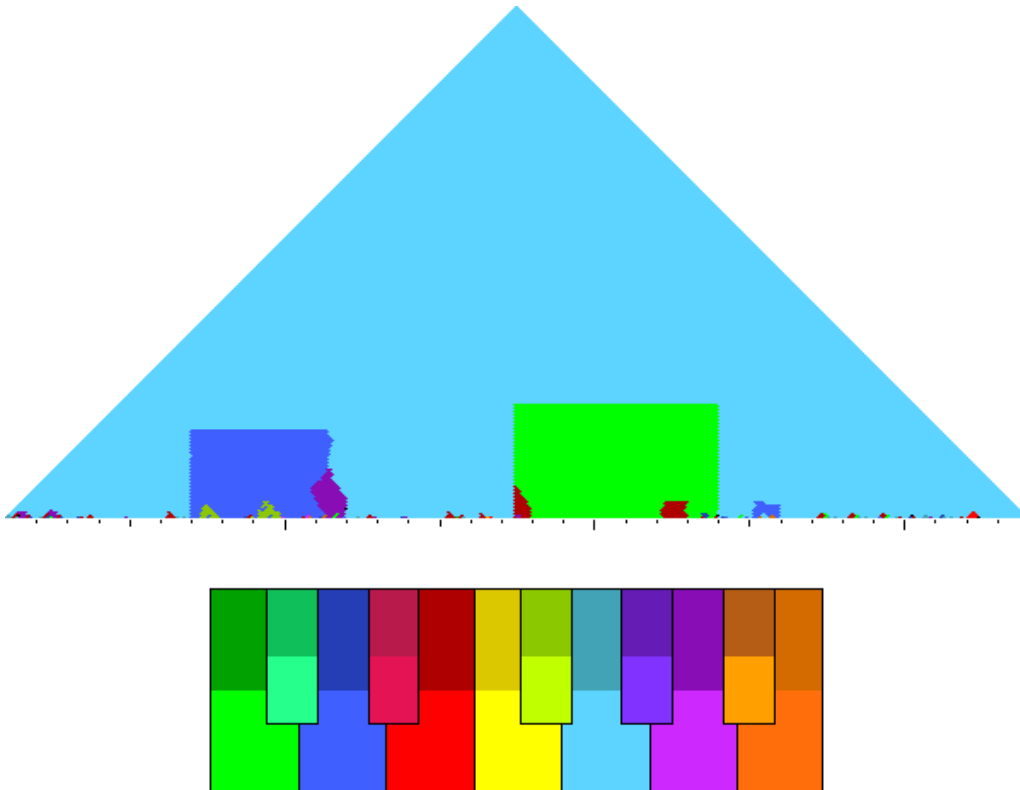


mkeyscape

<http://extras.humdrum.org/man/mkeyscape>

mkeyscape == “MIDI-aware keyscape program” (can read MIDI files and Humdrum files)

```
mkeyscape h://chopin/preludes/prelude28-03.krn -n --trim -l --aa | convert - z.png
```



-n == show measure ticks
--trim == post clean-up
-l == show color mapping legend
--aa == Use Aarden-Essen weights

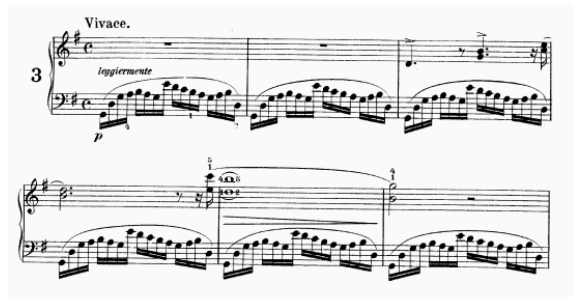
convert == ImageMagick image processing program (mkeyscape output is type PPM)

Chopin Prelude #3, Op. 28

G major

D major

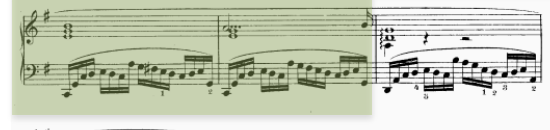
C major



D major

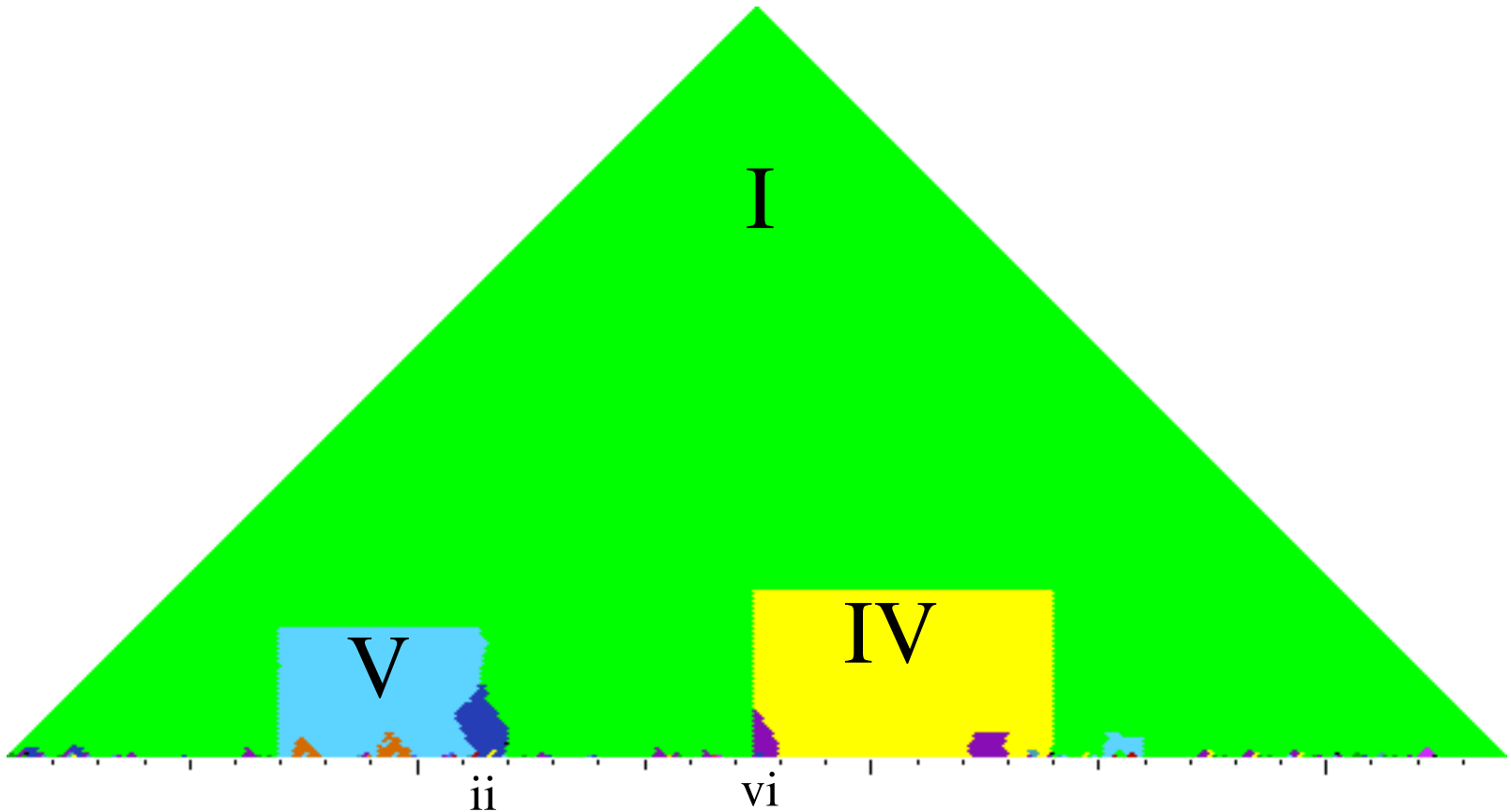


C major



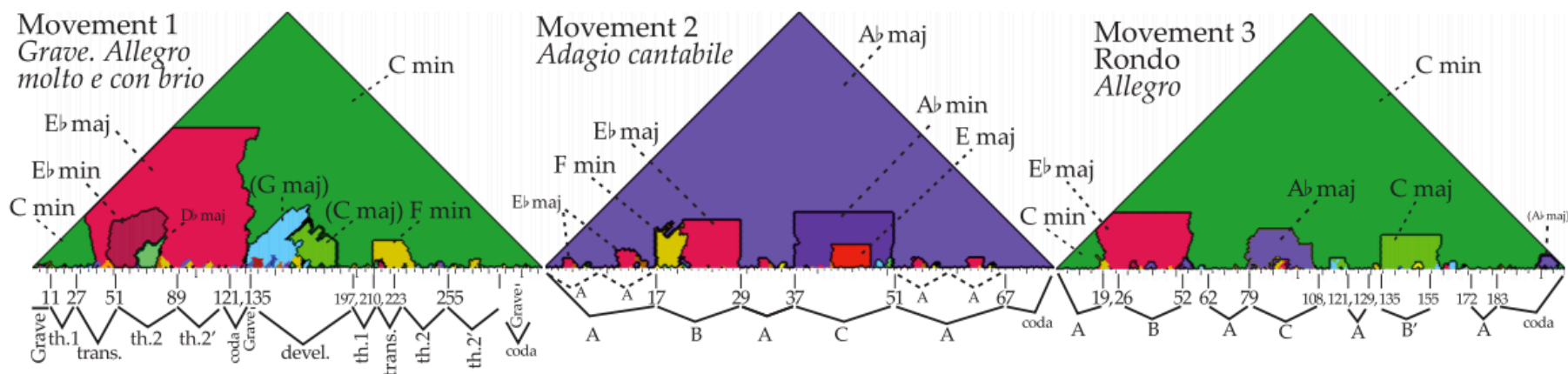
Functional Coloring

`transpose -k C h://chopin/preludes/prelude28-03.krn | mkeyscape -n --trim --aa | convert - z.png`



Sonata movements

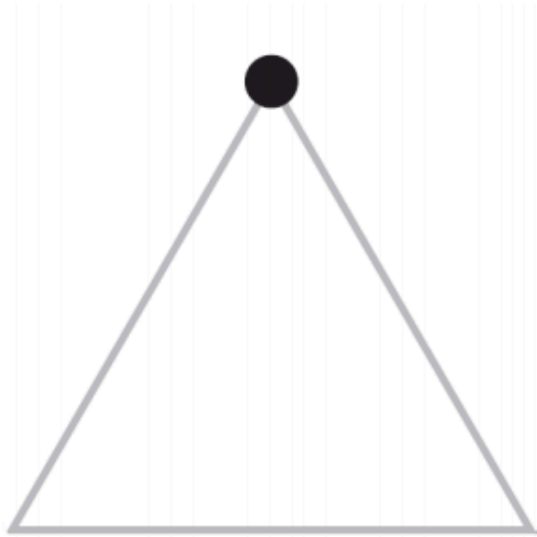
Ludwig van Beethoven Piano Sonata no. 8 in C minor, op. 13 ("*Pathétique*")



<http://extras.humdrum.org/man/mkeyscape/beet-sonata>

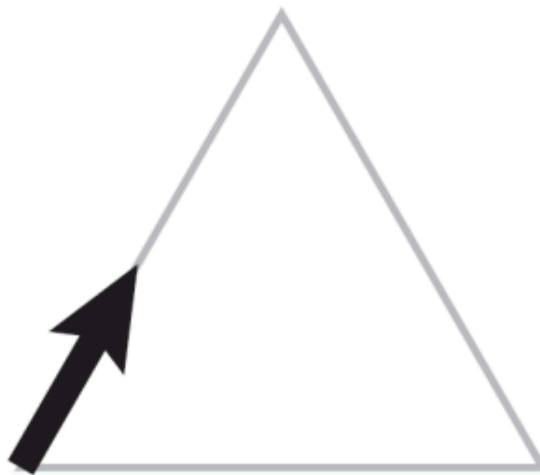
<http://extras.humdrum.org/man/mkeyscape/moz-sonata>

Key evaluation methods



Top-Level Test

high-data behavior



Leading-Edge Test

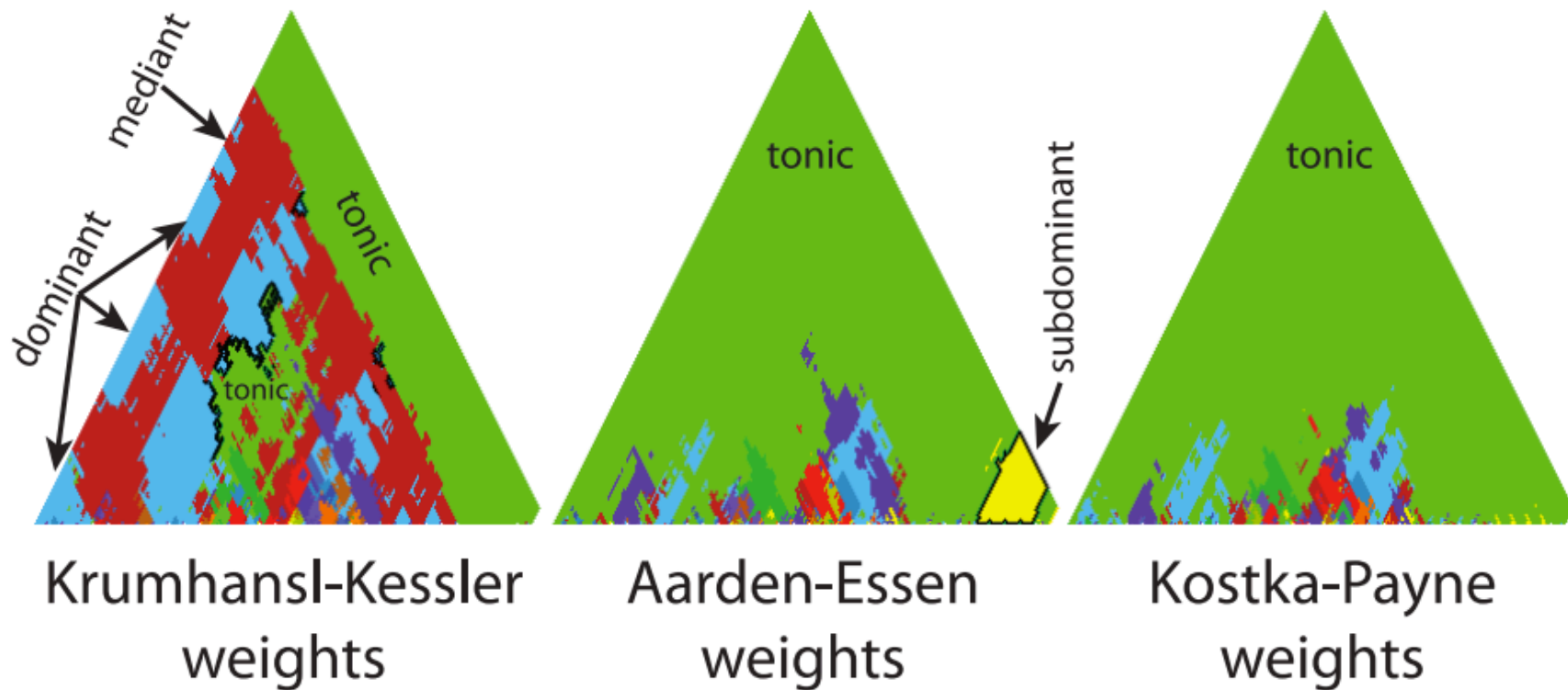
low-data behavior



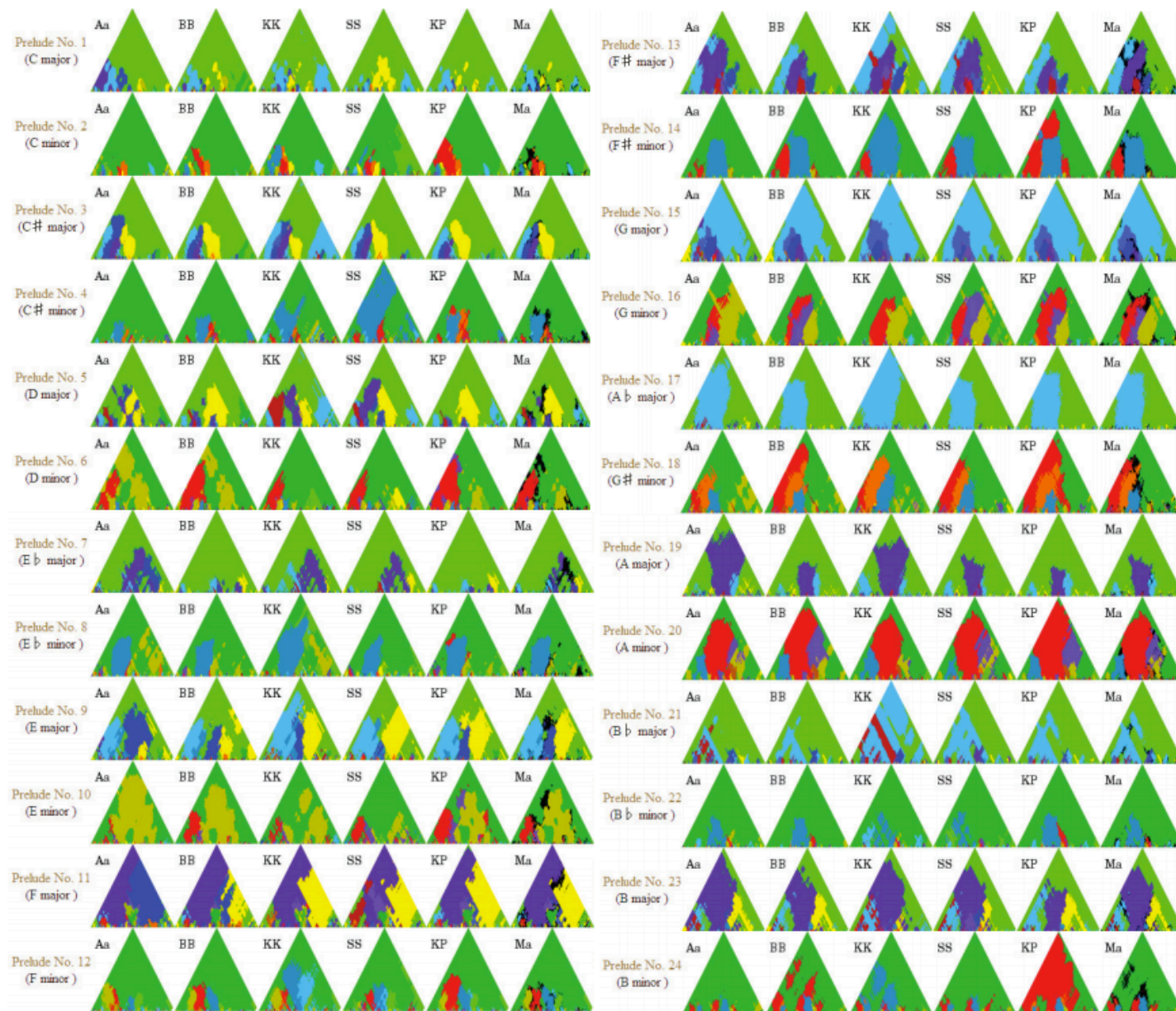
Tonic-Area Test

intermediate behavior

Keyscapes by weight



Keyscapes of WTC Book I Preludes



Tonic-area Evaluation

	J.S. Bach, The Well-tempered Clavier														Chopin preludes															
	Book I						Book II						op. 28																	
	preludes			fugues			preludes			fugues			preludes			fugues			preludes			fugues								
key	AE	BB	KK	KP	S	M	AE	BB	KK	KP	S	M	AE	BB	KK	KP	S	M	AE	BB	KK	KP	S	M	AE	BB	KK	KP	S	M
C maj	74	74	70	82	74	80	43	59	32	80	43	46	20	23	26	26	24	22	85	88	77	92	83	87	84	83	37	87	85	86
c min	87	85	76	74	65	81	88	81	82	36	83	83	14	11	18	10	16	13	90	91	90	72	90	90	83	86	80	83	77	82
C#maj	63	67	50	68	68	67	65	73	31	75	67	69	79	82	34	81	72	79	92	91	85	94	88	92	39	20	2	24	29	23
c#min	86	81	64	73	37	75	89	78	88	37	88	87	65	47	55	29	60	56	84	51	57	10	61	57	80	85	39	81	72	80
D maj	64	63	50	68	54	62	60	77	74	76	79	75	48	62	34	61	53	54	30	76	37	83	70	67	54	77	41	78	72	71
d min	47	51	75	56	70	62	84	86	71	79	73	82	75	76	56	59	77	73	90	90	89	77	89	89	55	55	32	45	43	46
E♭maj	64	86	67	88	76	76	46	71	50	88	52	56	65	86	49	84	78	78	90	91	46	89	85	89	72	73	60	77	74	73
e♭min	56	72	33	64	71	65	78	75	80	26	83	76	61	43	37	27	57	47	38	25	30	9	65	31	59	60	59	52	42	55
E maj	38	51	28	39	41	39	53	74	46	89	62	65	52	68	45	65	66	62	70	83	37	92	57	72	11	13	8	14	41	12
e min	34	46	61	48	71	49	26	11	26	7	13	13	60	51	34	26	59	51	73	46	62	19	59	57	48	65	49	67	81	62
F maj	3	7	6	7	8	6	24	44	12	67	26	31	10	33	12	59	25	24	67	73	67	79	74	73	82	84	38	77	65	77
f min	79	79	53	70	71	74	77	61	56	19	70	62	26	28	36	24	33	29	75	66	51	25	54	57	68	71	48	69	67	68
F#maj	41	57	27	59	43	45	34	63	35	69	50	50	40	65	28	72	50	51	40	79	56	79	70	69	85	86	75	89	84	87
f#min	73	62	46	45	64	60	90	87	82	36	84	85	70	64	63	46	76	67	82	42	76	3	80	74	44	65	19	55	32	46
G maj	18	23	16	21	25	21	32	52	32	63	53	51	62	74	27	70	65	67	43	69	8	77	56	55	61	53	46	63	58	58
g min	46	56	51	51	40	48	65	52	71	23	68	62	78	76	70	64	73	74	56	23	59	10	71	49	31	46	37	47	30	38
A♭maj	45	65	35	57	64	56	44	78	47	81	71	70	32	57	30	55	43	44	78	81	77	78	77	79	23	24	34	33	51	32
a♭min	59	46	46	38	54	50	71	64	50	10	72	65	40	38	25	26	32	33	93	88	85	22	87	89	56	58	54	52	55	56
A maj	40	75	48	78	74	71	53	86	63	89	78	79	50	59	67	69	60	62	19	65	11	71	37	38	86	87	61	85	84	86
a min	34	22	38	18	29	27	69	27	57	4	66	55	73	72	56	56	62	67	77	67	79	22	75	75	16	24	17	20	16	21
B♭maj	74	76	20	69	58	69	29	54	28	72	37	39	58	73	41	75	58	66	58	81	63	79	70	72	22	24	27	31	30	28
b♭min	81	84	78	78	79	82	37	32	70	24	65	40	52	33	51	23	50	45	79	38	73	9	61	64	67	72	60	49	69	66
B maj	25	40	31	52	38	37	70	84	54	84	77	79	45	65	20	61	50	52	25	82	59	88	66	72	94	94	68	93	90	92
b min	84	69	66	25	83	74	78	53	68	7	71	67	41	28	35	21	51	35	53	15	23	5	36	21	68	83	77	74	66	73

Tonic-area Evaluation (2)

= best result
 = better than simple weights

<i>weight set:</i>		AE	BB	KK	KP	SS	Majority
Major keys:	mean	0.70	0.90	0.58	0.96	0.83	0.82
	median	0.73	0.96	0.57	1.00	0.88	0.88
	sd	0.22	0.14	0.23	0.11	0.15	0.16
Minor keys:	mean	0.91	0.81	0.81	0.56	0.87	0.84
	median	0.99	0.88	0.84	0.61	0.93	0.87
	sd	0.14	0.21	0.17	0.30	0.14	0.13
Major and Minor combined:	mean	0.81	0.86	0.69	0.76	0.85	0.83
	median	0.91	0.93	0.72	0.90	0.90	0.87
	sd	0.21	0.18	0.23	0.30	0.14	0.14

More Info

<http://purl.stanford.edu/br237mp4161>

Lots of keyscares (raw plots; not post-processed):

<https://ccrma.stanford.edu/~craig/keyscape/class>

